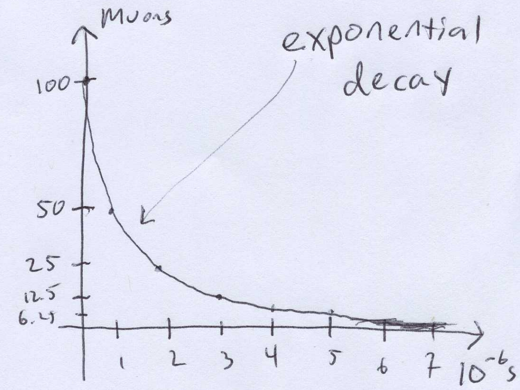
HW4 Solutions

HW4

(3) a) $\tau_\mu = 10^{-6}s$

The half-life is the time, on average, for your muon supply to reduce by $\frac{1}{2}$. Some points for the graph are

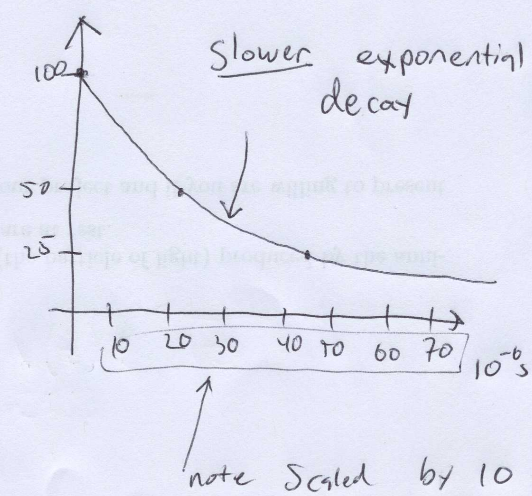
(100, 0s)
(50, τ_μ)
(25, $2\tau_\mu$)
(12.5, $3\tau_\mu$)
(6.25, $4\tau_\mu$)
(3.125, $5\tau_\mu$) →



b) $V = .999c$

$$(\tau_\mu)_{Lab} = \frac{(\tau_\mu)}{\sqrt{1-\frac{v^2}{c^2}}} \simeq 22\tau_\mu$$

(100, 0s)
(50, $22\tau_\mu$)
(25, $44\tau_\mu$)
(12.5, $88\tau_\mu$)



2. (Optional)

In the candle's reference frame the period between peak's will be $T_{\texttt{candle}} = 1/f$. Suppose the candle is moving away from you at a speed V. From the effect of time dilation we know the period between peaks will be $\delta t_1 = \gamma T_{\texttt{candle}}$. During the time interval δt_1 the candle has moved a distance $d = v\delta t_1$. Since light travels at a finite speed c, the light emitted at the second peak must travel an additional distance d to reach you, which adds an extra time-delay of $\delta t_2 = d/c$. Hence, you observe the period between peaks to be

$$\begin{aligned} T_{\texttt{you}} &= \delta t_1 + \delta t_2 \\ &= \gamma T_{\texttt{candle}} + \frac{v\gamma T_{\texttt{candle}}}{c} \\ &= T_{\texttt{candle}}\left[\gamma\left(1+\frac{v}{c}\right)\right] \\ &= T_{\texttt{candle}}\frac{1+\frac{v}{c}}{\sqrt{1-v^2/c^2}} \\ &= T_{\texttt{candle}}\frac{\sqrt{(1+\frac{v}{c})^2}}{\sqrt{(1+v/c)(1-v/c)}} \\ &= T_{\texttt{candle}}\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \end{aligned}$$

The Doppler shift formula becomes

$$f_{\texttt{you}} = f_{\texttt{candle}}\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

3. (Optional)

(i): find the dimensions of $G^{(d)}$. Notice that in both 3 and d spatial dimensions forces have the same units, and so $[F] = [G^{(d)}\frac{mM}{r^{d-1}}]$ and $[F] = [G^{(3)}\frac{mM}{r^2}]$ have the same units. From this observation we write

$$[G^{(d)}] = [G^{(3)}][\frac{r^{d-1}}{r^2}] = [G^{(3)}][r^{d-3}]$$

Notice that each extra spatial dimension beyond 3 adds a factor of meters to the units of $G^{(d)}$. Next we note that

$$[r^{d-3}] = [G^{(d)}]/[G^{(3)}]$$

follows from the previous equation, which suggests that the combination of $G^{(d)}$ and $G^{(3)}$ to relate with L is

$$L^{d-3} = G^{(d)}/G^{(3)}$$

(ii): recall the 3 dimensional plank length $\left(L_p^{(3)}\right)^2 = \frac{G^{(3)}h}{c^3}$. If we want to replace $G^{(3)}$ by $G^{(d)}$ in this expression we must add factors of meters to the left hand side. This discussion leads to an expression

$$\left(L_p^{(d)}\right)^{2+(d-3)} = \frac{G^{(d)}h}{c^3}$$

(iii): From step 2 we know

$$\frac{h}{c^3} = \frac{\left(L_p^{(d)}\right)^{d-1}}{G^{(d)}}$$

therefore

$$\left(L_p^{(3)}\right)^2 = \frac{G^{(3)}h}{c^3} = \frac{G^{(3)}}{G^{(d)}}\left(L_p^{(d)}\right)^{d-1}$$

The final result is

$$\frac{G^{(d)}}{G^{(3)}} = L^{d-3} = \frac{\left(L_p^{(d)}\right)^{d-1}}{\left(L_p^{(3)}\right)^{2}}$$