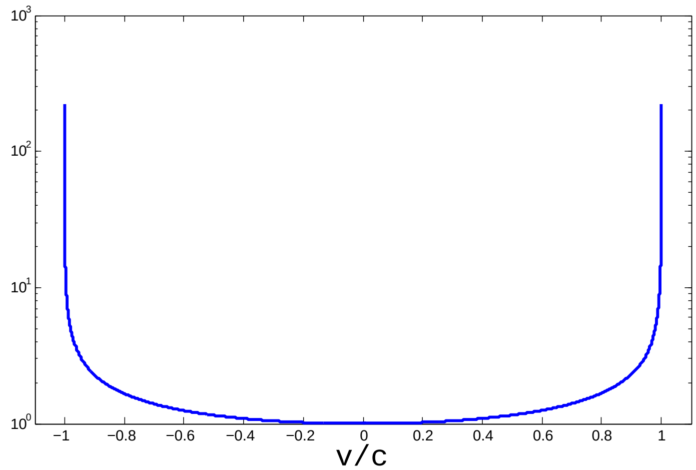
# HW3 Solutions

**1.** (a) $\gamma = \frac{1}{\sqrt{1-(v^2/c^2)}}$ is plotted in the figure below. Notice that the plot is (i) symmetric about $v = 0$, (ii) becomes infinitely large as $v \to c$ (equivalently $v/c \to 1$), (iii) the plot is more easily depicted with $v/c$ on the x-axis and using a logarithmic scale on the y-axis. Plots which did not use these choices are perfectly acceptable too.




(b) The problem becomes easier if we let $x = v/c$, and then $\gamma = \frac{1}{\sqrt{1-x^2}}$. Solving for $x$ I find

$$x = \pm\sqrt{1 - \frac{1}{\gamma^2}}$$

and so

$$v = \pm c\sqrt{1 - \frac{1}{\gamma^2}}\,.$$

Plugging in values for $\gamma$ we have $|v| \approx .86c$ when $\gamma = 2$, $|v| \approx 0.9950c$ when $\gamma = 10$, and $|v| \approx 0.9999c$ when $\gamma = 100$.

**2.** (a) $\left(\frac{99.9Km}{\text{hour}}\right)\left(\frac{1hr}{3600\text{s}}\right)\left(\frac{1000m}{1\text{km}}\right) = 2.77m/s$ and so $v/c \approx 10^{-8}$ and $\gamma \approx 1.0000000000000000005$. From the clerk's point of view you are 10 Km away traveling at 99.9 km/hour, so the total trip time (as seen by the clerk) is about 6 minutes. Your clocks duration will be $\frac{6}{\gamma}$ minutes, and so your clock is slow by $\frac{6}{\gamma} - 6 \approx 0$ minutes. Jumping right to the answer (not slower) is OK too, and encouraged.

(b) Now the speed of light is 100 km/hour and $\gamma \approx 22$. Your clocks duration will be $\frac{6}{22}$ minutes, so your watch is slow by $\frac{6}{22} - 6 \approx -5.7$ minutes.

**3.** Simply travel in a train or car at speeds close to 100 km/hour, within a few minutes (according to your watch) apply the breaks. It will be Friday. Unfortunately there is no way to come back to Wednesday (no backwards in time travel).

**4.** (Optional)

(a)

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{1}$$

(b) Directly from the equations is straightforward: plug in the definitions of $x'$ and $y'$ in terms of $x$ and $y$, then use trig properties. With Matrices let

$$M = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad V = \begin{pmatrix} x \\ y \end{pmatrix} \quad V' = \begin{pmatrix} x' \\ y' \end{pmatrix} \tag{2}$$

Then the equation in part a is $V' = MV$. And $(x')^2 + (y')^2 = (V')^T V'$ where $(V')^T$ is the transpose of a vector. Finally, showing that $(V')^T V' = V^T M^T M V = V^T V$ comes down to showing $M^T M$ has 1 on the diagonal and 0 on the off-diagonal. This can be shown by carrying out the matrix-matrix project and using $\sin^2 + \cos^2 = 1$.

(c) The Lorentz transformations given in class can be written as

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\frac{v}{c}\gamma & 0 & 0 \\ -\frac{v}{c}\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \tag{3}$$

which you can explicitly check to be true.

(d) Notice that $y'^2 = y^2$ and $z'^2 = z^2$ so theres nothing interesting to do for those coordinates. For the remaining two compute

$$\begin{aligned} x'^2 + (ct')^2 &= \left[\gamma\left(x - \frac{v}{c}ct\right)\right]^2 - \left[\gamma\left(ct - \frac{v}{c}x\right)\right]^2 \\ &= \gamma^2\left[x^2\left(1 - \frac{v^2}{c^2}\right) - (ct)^2\left(1 - \frac{v^2}{c^2}\right)\right] \\ &= x^2 - (ct)^2 \end{aligned}$$

where I've skipped over some of the algebraic steps.

Since one can write the Lorentz transformations as a matrix-vector ("linear") equation, and a kind-of distance is preserved, we conclude by analogy with more familiar spatial rotations that this is a rotation in spacetime. More closely mimicking the spatial rotations example done in parts a and b would require the knowledge of tensors and hyperbolic sine and cosine. The entry on wikipedia is pretty good for this: `http://en.wikipedia.org/wiki/Lorentz_transformation#Rapidity`.