HW3

Reading assignment: Mr Tompkins chapter 1 ("City Speed Limit")
Reading assignment: Feynman sections 3.3, 3.4, 3.5, 3.6
Please show all work!

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1. Sketch a curve of the relativistic factor γ from $v = -c$ to $v = c$. Plot γ on the vertical axis and v on the horizontal axis. What do you notice? When are special relativistic effects important: when is γ bigger than 2, 10, and 100?

2. Suppose you drive to the store located $10km$ away (as measured by the store clerk) at a speed $v = 99.9km/hour$ (recall speed of light is where $c = 3 \times 10^8 m/s$). Your watch and the store clerk's watch are perfectly synchronized before you head off. (a) provide an order of magnitude estimate for how much slower your watch is running (relative to the store clerk's watch) when you arrive at the store. (b) If the speed of light were $c = 100km/hour$, estimate how much slower your watch is running relative to the store clerk? (Hint: To answer this question think of the situation from the store clerk's point of view. Reading Mr. Tomkins will be very helpful towards gaining intuition.).

3. Suppose you find yourself in Mr. Tomkin's dream where the speed of light is $100km$ per hour. Its Monday and you really want to see a movie coming out on Friday. With words explain how you would time-travel *forward* in time so that you can effectively skip over Tuesday, Wednesday, and Thursday. You then realize you've missed a friend's birthday on Wednesday, is it possible to travel back to Wednesday? Why or why not?

3. (OPTIONAL – need vectors. This is a really good problem, take a few days if necessary.) Read Feynman Chapter 3.7. Feynman indicates that the Lorentz coordinate transformation is analogous to a rotation of space and time. In general, rotations can always be written as a matrix vector product and preserves a length measurement. (a) Write the equations $x' = x \cos\theta + y \sin\theta$ and $y' = y \cos\theta - x \sin\theta$ as a matrix vector product. (b) Either using properties of matrices, or directly from the equations, show that $x^2 + y^2 = (x')^2 + (y')^2$. We see that the length measurement is preserved. (c) Write the Lorentz transformation relating two observers K and K' in relative motion along the $x-$axis as a $4D$ matrix vector product relating the vectors $V' = [ct', x', y', z']^T$ and $V = [ct, x, y, z]^T$ (the symbol T denotes a vector transpose) by $V' = MV$ (c) Find the spacetime distance $x^2 + y^2 + z^2 - (ct)^2$ in terms of primed coordinates. What do you notice? From this observation deduce that Lorentz transformations can be viewed as a rotation in spacetime. If you are the K' observer, how would you "rotate back" to K?