HW1

- Reading assignment: Einstein chapters 1-6
- For the problems below, please show all work!
- Reading assignment (optional): Road to Reality chapter 1 handout.

---

1. Newton's law of gravitation is given by the equation below. F is the gravitational force between two objects of mass M and m which are separated by a distance r. Force has units of kg$\times\frac{m}{s^2}$. What are the units for the constant G?

$$F = G\frac{Mm}{r^2}$$

2. The speed of light is about $3 \times 10^8 m/s$. (a) Convert this to miles per hour. (b) If the sun is about 93×10^6 miles from Earth, provide an order of magnitude estimate for how long it takes the sun's light to reach Earth.

3. Quantum gravity is any theory which (correctly) describes the gravitational force using quantum mechanics. It has been called the Holy Grail of theoretical physics. Instead of quantum gravity we shall use the equation in problem 1 to describe gravitational interactions. While we won't discuss quantum mechanics, Plank's constant $\hbar$, which is given in units kg$\times m^2/s$, characterizes phenomena for which quantum mechanics is important.

Without knowing anything about quantum mechanics or quantum gravity we can use dimensional analysis to define the so-called Plank length and Plank time. (a) Using the constants $\hbar$, speed of light c, and the gravitational constant G (and nothing else!) try to find a combination which yields units of length and time. (b) Using the internet, find numerical values for these constants and provide an order of magnitude estimate for the Plank length and Plank time. Physical phenomena characterized by these scales, for example the big bang and black hole evaporation, require a quantum theory of gravity. (Hint: first compute the dimensions of $G\hbar/c^3$. What do you notice?)

4. In chapter 2 Einstein discusses different ways of measuring the distance to a cloud floating directly above Trafalgar square in London. Let us focus on two methods of measurement. Method A: build a coordinate system consisting of equally spaced markers and measure distance by counting the number of markers between the ground and the cloud. Method B: locate the cloud and directly measure the distance with, say, a very long ruler.

While these two approaches might seem similar they are actually a bit different. To understand why, suppose the cloud was initially 1000 meters above your head and then drifts 100 meters to your right. (a) Compute the distance from you to the cloud by method A. As part of your answer, draw the two dimensional coordinate system (known as a Cartesian coordinate system) by laying down rulers from the ground up and to the right. Find the clouds' distance from Trafalgar square by using the Pythagorean theorem. (b) Describe an experiment to compute the distance by method B.

5. **OPTIONAL – HARDER** Remember you are always free to choose your own coordinate system! A smarter choice of coordinate system for measuring the drifted cloud in problem 4 would be one in which you lay down equally spaced markers directly connecting Trafalgar square and the cloud. Taking Trafalgar square to be the origin of the coordinate system, relate this smarter coordinate system to the one you constructed in problem 4 by a rotation about the origin. Write down how the coordinates (i.e. x and y values) of these two systems are related. Finally, use this relationship to show distance measurements in either coordinate system will agree.

   You have "discovered" a fundamental property of the Universe: observers (i.e. people using different coordinate systems) related by rotation will agree on distances between two points. When we come to talk about special relativity we will see distance is NOT preserved when two observers are moving relative to one another.