HW5 Solutions

1. The masses of an electron and positron are both $M_e = 9 \times 10^{-31}$ Kg. If they are at rest the initial energy is

$$E_i = 2M_ec^2 \approx (18 \times 10^{-31}Kg)(9 \times 10^{16}m^2/s^2) \approx 10^{-13}J$$

By conservation of mass-energy $E_i = E_f$.

How does E_f relate to each individual photon's energy E_1 and E_2? By conservation of momentum each photon's momentum is equal and opposite amounts (because the initial momentum is zero). Since the magnitude of relativistic momentum of photon 1 and 2 are equal $P_1 = P_2 = P$ so are their energies $E_1 = E_2 = E$ (recall for massless particles $E_1 = P_1c$ and $E_2 = P_2c$). The final energy is then $E_f = E_1 + E_2 = 2Pc$. Conservation of mass-energy ($E_i = E_f$) then gives

$$E_i = 10^{-13}J = E_f = 2Pc$$

The momentum of each photon is

$$P = \frac{10^{-13}J}{2 \times (3 \times 10^8 m/s)} \approx 1.67 \times 10^{-22}kgm/s$$

and the energy is

$$E = Pc = \frac{1}{2}10^{-13}J$$

2. Start by drawing a figure and label the angle made by the two lines meeting at the north pole by ϕ. Since the other two angles are 90 degrees each (equivalently $\pi/2$ each) the sum of Angles is

$$\text{Angles} = \pi + \phi$$

Next, recall that the area of a ball of radius r is $4\pi r^2$. So, as a special case, whenever $\phi = 2\pi$ the entire northern hemisphere of the ball is "covered" by our triangle – the triangle's area would be $2\pi r^2$. If $\phi = 0$ the area would be 0. With a little thought we see the triangle's area to be

$$\text{Area} = \frac{4\pi r^2}{2(2\pi/\phi)} = \phi r^2$$

From the angle and area formulas we have

$$\text{Angles} = \pi + \frac{\text{Area}}{r^2}$$

Notice that whenever r is very very large (for example Earth) we have the usual Euclidean formula Angles $= \pi$.