

HW1 SOLUTIONS

1. First rearrange the equation $G = \frac{Fr^2}{Mm}$ and using dimensional analysis we see

$$[G] = \left[\frac{Fr^2}{Mm} \right] = \frac{[F][r^2]}{[M][m]} = \frac{(kg \frac{m}{s^2}) m^2}{(kg)^2} = \frac{m^3}{s^2 kg}.$$

For this calculation we used the fact that $[F] = kg \frac{m}{s^2}$, $[r^2] = m^2$, and $[M] = [m] = kg$.

2. 1 hour = $60 \times (60s) = 3600s$

1 mile = 1609.3 meters

So in miles per hour the speed of light is

$$(3 \times 10^8 m/s) \left(\frac{1 \text{ mile}}{1609.3 \text{ meters}} \right) \left(\frac{3600s}{1 \text{ hour}} \right) \approx 6.7 \times 10^8 \text{ miles/hour}.$$

If the sun is about 93×10^6 miles from Earth it takes

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{93 \times 10^6 \text{ miles}}{6.7 \times 10^8 \text{ miles/hour}} \approx .139 \text{ hours},$$

or about 8 minutes for the sun's light to reach Earth.

3. The physical constants we are allowed to use have units of $[G] = \frac{m^3}{s^2 kg}$, $[c] = m/s$, and $[\hbar] = kg \times m^2/s$. Notice that $[G\hbar/c^3] = m^2$, and so $[G\hbar/c^5] = (m^2)(\frac{s^2}{m^2}) = s^2$. The Plank time and length are

$$L_{\text{Plank}} = \sqrt{G\hbar/c^3}$$

$$T_{\text{Plank}} = \sqrt{G\hbar/c^5}.$$

Using wikipedia I found the physical constants to be (in MKS units) roughly $G = 6 \times 10^{-11}$, $c = 3 \times 10^8$, $\hbar = 6 \times 10^{-34}$

$$L_{\text{Plank}} = \sqrt{G\hbar/c^3} \approx \sqrt{\frac{(6 \times 10^{-11})(6 \times 10^{-34})}{(3 \times 10^8)^3}} \approx \sqrt{\frac{36 \times 10^{-45}}{27 \times 10^{24}}} \approx 10^{-35}$$

$$T_{\text{Plank}} = \sqrt{G\hbar/c^5} \approx \frac{10^{-35}}{10^8} = 10^{-43}.$$

Because we are working in MKS units these values are in meters and seconds, respectively.

4. Place Trafalgar square at the origin of the coordinate system. Step 1: using, say, a 200 meter ruler measure equally space 'ticks' (denoted by the red stars) from the ground to the original location of the cloud and call this the y-axis. Step 2: Now switch to, say, a 50 meter ruler. Starting from each tick on the t-axis lay down equally spaced 'ticks' to the right (at a 90 degreeed angle from the x-axis). Step 3: continue the process until you have finished building your Cartesian coordinate system. See figure below. The Pythagorean theorem tells us $\text{Distance} = \sqrt{(\Delta X)^2 + (\Delta Y)^2} = \sqrt{(100m)^2 + (1000m)^2} \approx 1005m$.

5. In fact Einstein's measurement experiment 'b' and 'c' provide good evidence that these distances will be the same. A proof can be given by observing that the smarter coordinate system can be "built" by rotating the first coordinate system by an angle θ in the counterclockwise direction. From the figure below we can find the location of the cloud is given by $x' = x \cos \theta + y \sin \theta$ and $y' = y \cos \theta - x \sin \theta$. The distance of the cloud from Trafalgar square is EXACTLY the same in both coordinate systems

$$(x')^2 + (y')^2 = (x \cos \theta + y \sin \theta)^2 + (y \cos \theta - x \sin \theta)^2 = x^2(\cos^2 \theta + \sin^2 \theta) + y^2(\cos^2 \theta + \sin^2 \theta) = x^2 + y^2.$$

If the x' -axis is aligned with the cloud (so that $y' = 0$) you get $x' = \sqrt{x^2 + y^2} = \text{Distance}$. So the clouds location on the x-axis directly measures the distance.

