Theory of Relativity – Final Quiz
July 10, 2013

Name:____

Below are short questions and problems. Answer to the best of your ability.

VERY short answers. Each worth 1 point.

1. What is the principle of relativity? How many versions are there?

The laws of physics are the same in all inertial reference frames. There are three versions: Galilean, special and general.

2. A ship is moving near the speed of light and fires a laser beam. According to special relativity what will a stationary observer measure the laser beam's speed to be? what will an observer located on the ship measure the laser beam's speed to be?

Both will measure the speed of light to be $c = 3 \times 10^8$ m/s

3. Draw a spacetime diagram and label the causal structure of an observer located at the origin. On this diagram draw a worldine which is NOT allowed.

A good figure can be found here http://www.theculture.org/rich/sharpblue/archives/000089.html

Any curve whose slope is less than 1 works. For example the line connecting event P and Q (see the link).

4. Suppose I am in an inertial reference frame and you move at a speed $v = 10m/s$ relative to me. Are you an inertial observer?

Yes

5. List 3 ways our life would be different if the speed of light were $80km/hour$.

You would age appreciably less after a bike ride. Cars would shrink in the direction of motion. Can't talk of friends across the country in "real-time". Can't go faster than 80.

6. What is the resolution to the twin paradox?

Suppose twin A accelerates to compare watches. This twin is no longer an inertial observer (they know they are moving). The other twin (B) is always inertial and so their calculations are correct.

7. Suppose the Michaelson-Morley experiment discovered aether. If the aether wind moved at exactly the speed of light and you fired a laser against the aether wind what would light's speed be? (Hint: this is before special relativity physics, think of the airplane problem).

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8. Why does the Foucault pendulum appear to rotate throughout the day?

Earth is rotating on its axis - Earth is a non-inertial reference frame.

9. According to special relativity, if you continue to push a ball by applying a constant force at some point you can hardly increase the velocity, yet you continue to do work on the ball. Where does this extra energy go?

Into its inertial mass.

10. A consequence of the equivalence principles is that light bends in a gravitational field. What startling suggestion did this lead Einstein to make about our spacetime geometry?

Its non-Euclidean (curved spacetime).

11. Suppose you take a long voyage to a distant star and return home. You are paid at an hourly rate. Will you be paid more going by the Earth's clock or the ship's clock?

Earth

12. A rocket is moving away from you. According to special relativity, compared to its length on the ground you will measure its length to be (a) shorter, longer, or the same? (b) Will your answer change if the rocket comes towards you.

(a) shorter (b) no

Problems – Optional. Each worth 2 points

1. In Back to the Future 4 (the unreleased sequel) Marty goes 500 seconds into Earth's future in what seems to be 300 seconds to him by moving at $v = 140km/hour$. According to special relativity what would the speed of light need to be in the Back to the Future universe for this to happen? You can leave your answer expressed as some number times v.

From the time dilation formula we have

$$500s = \gamma(300s) \quad \rightarrow \quad \gamma = 5/3$$

Solving for the (Back to the future) speed of light

$$\begin{aligned} 5/3 = \gamma &= \frac{1}{\sqrt{1 - v^2/c^2}} \\ \sqrt{1 - v^2/c^2} &= 3/5 \\ 1 - v^2/c^2 &= 9/25 \\ v^2/c^2 &= 16/25 \end{aligned}$$

And so $c = 5/4v = 175km/hour = 48m/s$

2. Suppose Nolan Ryan could throw a fastball at $v = (4/5)c$. If a baseball's rest mass is $1kg$, what is the ball's mass while its in motion? What is its relativistic kinetic energy? How does this compare to the usual $\frac{1}{2}m_0v^2$ expression for the kinetic energy (greater, less equal)? For this problem you might need the formula $E = \gamma m_0c^2 = m_0c^2+$(relativistic kinetic energy).

$v = (4/5)c$, and so by the previous problem $\gamma = 5/3$. $M = m_0\gamma = 5/3$ kg.
Relativistic kinetic energy is $(m_0\gamma - m_0)c^2 = \frac{2}{3}m_0c^2$
Classical kinetic energy is $\frac{1}{2}m_0v^2 = \frac{1}{2}m_0(\frac{4}{5}c)^2 = \frac{8}{25}m_0c^2$
Relativistic kinetic energy is greater.

3. In $d \geq 3$ spatial dimensions Newton's gravitational law becomes $F = G^{(d)}\frac{mM}{r^{d-1}}$ where $G^{(d)}$ is a constant. This suggests gravity's weakness (compared to the other forces such as electromagnetism) might be due to the fact that we live in a higher dimensional world. (i) Using dimensional analysis find a formula for the d dimensional plank length for arbitrary d dimensions. (ii) suppose $d-3$ of these dimensions are compactified, that is they are wrapped up into a "circles" of length L, using dimensional analysis show how L can be related to some combination of G^3 and G^d. (iii) $L^{(d-3)}$ is the volume of the compactified dimensions. Using previously derived formulas, relate $L^{(d-3)}$ to some combination of the d-dimensional plank length and the 3-dimensional plank length. This relation enables us to explore the possibility that the world is actually d-dimensional with a fundamental Planck length that is much larger than 10^{-35} meters. Of course, the 3-dimensional Plank length is 10^{-35} meters. Particle accelerators have probed length's down to 10^{-20} meters with no signs of extra dimensions yet.

Step 1: find the dimensions of $G^{(d)}$. Notice that in both 3 and d spatial dimensions forces have the same units, and so $[F] = [G^{(d)} \frac{mM}{r^{d-1}}]$ and $[F] = [G^{(3)} \frac{mM}{r^2}]$ have the same units. From this observation we write

$$[G^{(d)}] = [G^{(3)}][\frac{r^{d-1}}{r^2}] = [G^{(3)}][r^{d-3}]$$

Notice that each extra spatial dimension beyond 3 adds a factor of meters to the units of $G^{(d)}$. Next we note that

$$[r^{d-3}] = [G^{(d)}]/[G^{(3)}]$$

follows from the previous equation, which suggests that the combination of $G^{(d)}$ and $G^{(3)}$ to relate with L is

$$L^{d-3} = G^{(d)}/G^{(3)}$$

Step 2: recall the 3 dimensional plank length $\left(L_p^{(3)}\right)^2 = \frac{G^{(3)}h}{c^3}$. If we want to replace $G^{(3)}$ by $G^{(d)}$ in this expression we must add factors of meters to the left hand side. This discussion leads to an expression

$$\left(L_p^{(d)}\right)^{2+(d-3)} = \frac{G^{(d)}h}{c^3}$$

Step 3: From step 2 we know

$$\frac{h}{c^3} = \frac{\left(L_p^{(d)}\right)^{d-1}}{G^{(d)}}$$

therefore

$$\left(L_p^{(3)}\right)^2 = \frac{G^{(3)}h}{c^3} = \frac{G^{(3)}}{G^{(d)}} \left(L_p^{(d)}\right)^{d-1}$$

The final result is

$$\frac{G^{(d)}}{G^{(3)}} = L^{d-3} = \frac{\left(L_p^{(d)}\right)^{d-1}}{\left(L_p^{(3)}\right)^2}$$