

HW3

Reading assignment: Mr Tompkins Handout.

Optional Reading assignment: Feynman sections 3.3, 3.4, 3.5, 3.6 (Mostly what we did in class).

Please do the first two problems below. There are two extra problems for those who like a challenge. Please show all work!

1. (a) Draw a curve of the relativistic factor γ for $v = -.99999c$ to $v = .99999c$ (γ on the vertical y-axis and v on the horizontal x-axis). Suppose you drive to the store located $10km$ away (as measured by the store clerk) at a speed $v = 99.9km/hour$ (recall speed of light is where $c = 3 \times 10^8 m/s$), (b) provide an order of magnitude estimate for how much slower your watch is running when you get to the store. (c) If the speed of light were $c = 100km/hour$, estimate how much slower your watch is running? (Hint: Time is relative! To answer this question think of the situation from the store clerk's point of view. Reading Mr. Tomkins will be very helpful towards gaining intuition.).
2. Suppose you find yourself in Mr. Tomkin's dream where the speed of light is $100km$ per hour. Its Monday and you really want to see a movie coming out on Friday. With words explain how you would time-travel *forward* in time so that you can skip over Tuesday, Wednesday, and Thursday. You then realize you've missed a friend's birthday on Wednesday, is it possible to travel backwards to Wednesday? Why or why not?
3. (OPTIONAL – need vectors. This is a really good problem, take a few days if necessary.) Read Feynman Chapter 3.7. Feynman indicates that the Lorentz coordinate transformation is analogous to a rotation of space and time. In general, rotations can always be written as a matrix vector product and preserves a length measurement. (a) Write the equations $x' = x \cos \theta + y \sin \theta$ and $y' = y \cos \theta - x \sin \theta$ as a matrix vector product. (b) Either using properties of matrices, or directly from the equations, show that $x^2 + y^2 = (x')^2 + (y')^2$. We see that the length measurement is preserved. (c) Write the Lorentz transformation relating two observers K and K' in relative motion along the x -axis as a $4D$ matrix vector product relating the vectors $V' = [ct', x', y', z']^T$ and $V = [ct, x, y, z]^T$ (the symbol T denotes a vector transpose) by $V' = MV$ (c) Find the spacetime distance $x^2 + y^2 + z^2 - (ct)^2$ in terms of primed coordinates. What do you notice? From this observation deduce that Lorentz transformations can be viewed as a rotation in spacetime.
4. (OPTIONAL – need calculus. Take a few days if necessary.) Following Feynman section 4.3, derive the relativistic addition of velocity formula.