HW1 Solutions

1. First rearrange the equation $G = \frac{Fr^2}{Mm}$ and using dimensional analysis we see

$$[G] = \left[\frac{Fr^2}{Mm}\right] = \frac{[F][r^2]}{[M][m]} = \frac{\left(kg\frac{m}{s^2}\right)m^2}{(kg)^2} = \frac{m^3}{s^2 kg}.$$

For this calculation we used the fact that $[F] = kg\frac{m}{s^2}$, $[r^2] = m^2$, and $[M] = [m] = kg$.

2. 1 hour $= 60 \times (60s) = 3600s$

1 mile $= 1609.3$ meters

So in miles per hour the speed of light is

$$(3 \times 10^8 m/s)\left(\frac{1\text{mile}}{1609.3\text{meters}}\right)\left(\frac{3600s}{1\text{hour}}\right) \approx 6.7 \times 10^8\text{miles/hour}.$$

If the sun is about 93×10^6 miles from Earth it takes

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{93 \times 10^6\text{miles}}{6.7 \times 10^8\text{miles/hour}} \approx .139\text{hours},$$

or about 8 minutes for the sun's light to reach Earth.

3. The physical constants we are allowed to use have units of $[G] = \frac{m^3}{s^2 kg}$, $[c] = m/s$, and $[\hbar] = kg \times m^2/s$. Notice that $[G\hbar/c^3] = m^2$, and so $[G\hbar/c^5] = (m^2)(\frac{s^2}{m^2}) = s^2$. The Plank time and length are

$$L_{\text{Plank}} = \sqrt{G\hbar/c^3}$$
$$T_{\text{Plank}} = \sqrt{G\hbar/c^5}.$$

Using wikipedia I found the physical constants to be (in MKS units) roughly $G = 6 \times 10^{-11}$, $c = 3 \times 10^8$, $\hbar = 6 \times 10^{-34}$

$$L_{\text{Plank}} = \sqrt{G\hbar/c^3} \approx \sqrt{\frac{(6 \times 10^{-11})(6 \times 10^{-34})}{(3 \times 10^8)^3}} \approx \sqrt{\frac{36 \times 10^{-45}}{27 \times 10^{24}}} \approx 10^{-35}$$
$$T_{\text{Plank}} = \sqrt{G\hbar/c^5} \approx \frac{10^{-35}}{10^8} = 10^{-43}.$$

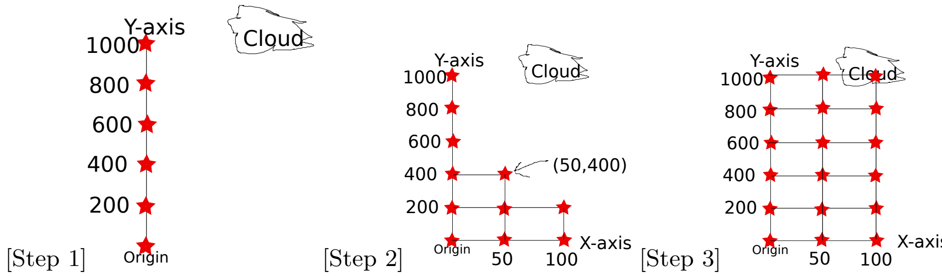
Because we are working in MKS units these values are in meters and seconds, respectively.

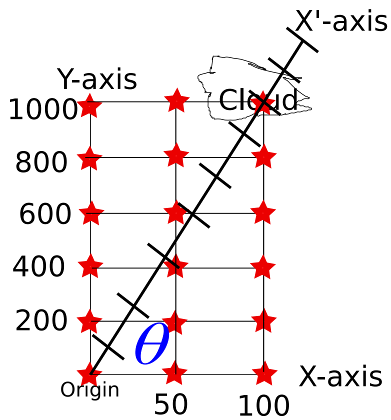
4. Place Trafalgar square at the origin of the coordinate system. Step 1: using, say, a 200 meter ruler measure equally space 'ticks' (denoted by the red stars) from the ground to the original location of the cloud and call this the y-axis. Step 2: Now switch to, say, a 50 meter ruler. Starting from each tick on the t-axis lay down equally spaced 'ticks' to the right (at a 90 degreed angle from the x-axis). Step 3: continue the process until you have finished building your Cartesian coordinate system. See figure below. The Pythagorean theorem tells us Distance $= \sqrt{(\Delta X)^2 + (\Delta Y)^2} = \sqrt{(100m)^2 + (1000m)^2} \approx 1005m$.

5. In fact Einstein's measurement experiment 'b' and 'c' provide good evidence that these distances will be the same. A proof can be given by observing that the smarter coordinate system can be "built" by rotating the first coordinate system by an angle θ in the counterclockwise direction. From the figure below we can find the location of the cloud is given by $x' = x\cos\theta + y\sin\theta$ and $y' = y\cos\theta - x\sin\theta$. The distance of the cloud from Trafalgar square is EXACTLY the same in both coordinate systems

$$(x')^2 + (y')^2 = (x\cos\theta + y\sin\theta)^2 + (y\cos\theta - x\sin\theta)^2 = x^2(\cos^2\theta + \sin^2\theta) + y^2(\cos^2\theta + \sin^2\theta) = x^2 + y^2.$$

If the x'-axis is aligned with the cloud (so that $y' = 0$) you get $x' = \sqrt{x^2 + y^2} =$ Distance. So the clouds location on the x-axis directly measures the distance.


[Step 1]
Y-axis
Cloud
1000
800
600
400
200
Origin
[Step 2]
Y-axis
Cloud
1000
800
600
400
(50,400)
200
Origin
50
100
X-axis
[Step 3]
Y-axis
Cloud
1000
800
600
400
200
Origin
50
100
X-axis


X'-axis
Y-axis
Cloud
1000
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X-axis