

THEORY OF RELATIVITY – FINAL QUIZ
JULY 7, 2011

Name: _____

Below are short questions and problems. Answer to the best of your ability. All equations and constants you need are on a separate sheet.

VERY short answers. Each worth 1 point.

1. According to special relativity, if a ship moving at $v = .99999c$ fires a laser beam what will a stationary observer measure the laser beam's speed to be? What does the ship measure the laser beam's speed to be?

Both to be c

2. The equation $E = mc^2$ is often given in popular descriptions of Special Relativity. Is it correct for moving masses?

No, $E^2 = (m_0c^2)^2 + (pc)^2$

3. According to special relativity, what two speed measurements will observers always agree on?

- ① Speed of light
② Relative speed between each other

4. Inertial observers in Newton's theory are related by what type of coordinate transformation (the name, although feel free to write the equations)? What coordinate transformation relates inertial observers in special relativity?

Galilean
Lorentz

5. Suppose I am in an inertial frame and you move at a speed $v = 10\text{m/s}$ relative to me. Are you an inertial observer?

Yes

6. List some ways our life would be different if the speed of light were 80km/hour .

length contraction

time dilation

can't go faster than 80 km/hr

7. What is the resolution to the twin paradox?

To compare clocks, one twin must accelerate towards the other, and so no longer inertial. We can also resolve by working with the

8. Suppose the Michaelson-Morley experiment discovered aether. If the aether wind moved at exactly the speed of light and you fired a laser into the aether wind what would light's speed be? (Hint: this is before special relativity physics, think of the airplane problem).

world line of the twin in motion.



9. You are describing the motion of a car. When will it become important to include special relativistic effects?

as $v \rightarrow c$

10. Why does the Foucault pendulum appear to rotate throughout the day?

Earth is spinning, and so is non-inertial

11. We observe many muons on Earth, yet the muon's half-life suggests those created in the upper atmosphere should not make it to sea level. How is this possible?

they move close to the speed of light, and so experience time dilation.

12. According to special relativity, if you continue to push a ball by applying a constant force at some point you can hardly increase the velocity, yet you continue to do work on the ball. Where does this extra energy go?

object's mass increases

13. Equivalence of inertial and gravitational mass was suggested long before Einstein. Einstein takes this a step further with the strong equivalence principle. What is strong equivalence principle?

No local measurement can distinguish between accelerated motion and a gravitational field

14. A consequence of the strong equivalence is that light bends in a gravitational field. What startling suggestion did this lead Einstein to make about our spacetime geometry?

Spacetime is curved

15. While sitting in this room taking this quiz, do you think your head or feet have aged quicker? Or neither?

head

16. Suppose you take a long voyage to a distant star and return home. You are paid at an hourly rate. Will you be paid more going by the Earth's clock or the ship's clock?

Earth

17. (BONUS) Nobody calls Marty McFly this name?

chicken

①

$$500 \text{ s} = \gamma(300 \text{ s})$$

$$\gamma = \frac{5}{3} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{9}{25} = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = \frac{16}{25}$$

$$c^2 = \frac{25}{16} v^2$$

$$c = \frac{5}{4} v = 175 \frac{\text{km}}{\text{hour}} \\ \approx 48 \text{ m/s}$$

(2) $v = \frac{4}{5}c$, so by the previous problem
 $\gamma = \frac{5}{3}$

$$m = m_0 \gamma = \frac{5}{3} m_0$$

$$KE_{SR} = m_0 \gamma c^2 - m_0 c^2 = \left(\frac{5}{3} - 1\right) m_0 c^2 = \frac{2}{3} m_0 c^2$$

$$KE_{classical} = \frac{1}{2} m_0 v^2 = \frac{1}{2} m_0 \left(\frac{4}{5}c\right)^2 = \frac{8}{25} m_0 c^2$$

$$KE_{classical} < KE_{SR}$$

③ Relativistic Doppler Shift

$$\frac{E}{c} = \gamma \left(\frac{E'}{c} - \frac{v}{c} p' \right)$$

$$p = \gamma \left(p' - \frac{v}{c} \left(\frac{E'}{c} \right) \right)$$

→ We only need is equation and
 $E' = p'c \rightarrow p' = \frac{E'}{c}$

$$\frac{E}{c} = \gamma \left(\frac{E'}{c} - \frac{v}{c^2} E' \right)$$

$$E = \gamma \left(E' - \frac{v}{c} E' \right) = \gamma \left(1 - \frac{v}{c} \right) E'$$

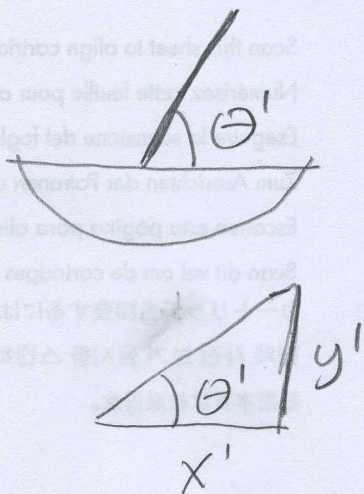
and so

$$E' = \frac{\gamma^{-1} E}{1 - \frac{v}{c}} = \left[\frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c}} \right] E$$

use $E = hf$ $E' = hf'$ to conclude

$$f' = \left[\frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c}} \right] f$$

(4)



$$\tan \theta' = \frac{y'}{x'}$$

Lorentz contraction in x only

$$y = y'$$

$$X = \frac{X'}{\gamma} \Rightarrow X' = \gamma X$$

and so

$$\tan \theta' = \frac{y'}{x'} = \frac{y}{\gamma X} = \frac{1}{\gamma} \left(\frac{y}{X} \right) = \gamma^{-1} \tan \theta$$

so,

$$\tan \theta = \gamma \tan \theta'$$

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \approx 2.3$$

$$\tan \theta = (2.3) \tan \theta'$$