Non-perturbative self-force effects and improving perturbation theory

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15th Capra Meeting
University of Maryland
June 13, 2012

Overview

- ☐ Survey of nonlinear scalar self-force
- ☐ Nonperturbative self-force effects
- ☐ Improving perturbation theory

Why bother with higher order? (1)

- The more accurately the waveform can be calculated, the more accurately the parameters can be measured

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- Transient resonances may effect parameter estimation

$$\Phi \sim \frac{1}{\varepsilon} + \frac{1}{\sqrt{\varepsilon}} + O(\varepsilon^0)$$

Flanagan & Hinderer (2010)

- Change of phase by about 15 rad if waveform doesn't track resonance

Why bother with higher order? (2)

- Could describe binaries with intermediate mass ratios and possibly with comparable mass components

See Alexandre Le Tiec's talk

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- ☐ Others?...

Why bother with scalar fields?

- Historically, scalar models offer a simpler framework

  - Most useful regularization scheme (Detweiler & Whiting (2003)) first developed and understood in a scalar model

  - Numerical self-force computations first accomplished for linear scalar models

  - Conceptually cleaner because not a gauge theory

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- Higher order perturbative expressions can be used "out of the box" for Green function based self-force computations

  - Hadamard decomposition most useful for this

  - Applicable for arbitrary accelerated orbits

Nonlinear scalar self-force:

A brief survey

CRG, CQG (2012a, b)

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  - *The most interesting ones are "similar" to perturbed General Relativity*

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$$\Downarrow$$

$$\boxed{S[z^\mu, \phi] = -\sum_{n=2}^{\infty}\frac{a_n}{n!}\int_x \nabla_\alpha\phi\nabla^\alpha\phi\,\phi^{n-2} - m\sum_{n=0}^{\infty}\frac{b_n}{n!}\int d\tau\,\phi^n(z^\alpha)}$$

Nonlinear scalar theory (2)

- A fortuitous change of field variable removes all self-interactions of the field

$$\nabla_\alpha \psi = \nabla_\alpha \phi \left(1 + \sum_{n=1}^{\infty} \frac{2a_{n+2}}{(n+2)!} \phi^n \right)^{1/2}$$

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$$\boxed{S[z^\mu, \psi] = -\frac{1}{2} \int_x \nabla_\alpha \psi \nabla^\alpha \psi - m \sum_{n=0}^{\infty} \frac{c_n}{n!} \int d\tau \, \psi^n(z^\alpha)}$$

- Action for a nonlinear scalar field becomes one for a linear field with nonlinear couplings to the small body

Self-force and radiated field (1)

- In previous work, the 3rd order self-force and corresponding radiation field were derived *CRG, CQG (2012a, b)*

  - Calculated using EFT and the classical mechanics for non-conservative systems

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$$ma^\mu = -m(g^{\mu\nu} + u^\mu u^\nu)\nabla_\nu \ln C\big(\psi_R(z^\mu)\big) \qquad C\big(\psi_R(z^\mu)\big) \equiv \sum_{n=0}^{\infty} \frac{c_n}{n!}\, \psi_R(z^\mu)$$

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  - *Regular part of field calculated through 3rd order with DW decomposition*

$$\begin{aligned}\psi_R(z^\mu) = & -mc_1 I_R(z^\mu) + m^2 c_1 c_2 \int d\tau'\, G_R(z^\mu, z^{\mu'}) I_R(z^{\mu'}) \\ & - m^3 c_1 c_2^2 \int d\tau' \int d\tau''\, G_R(z^\mu, z^{\mu'}) G_R(z^{\mu'}, z^{\mu''}) I_R(z^{\mu''}) \\ & - \frac{m^3 c_1^2 c_3}{2} \int d\tau'\, G_R(z^\mu, z^{\mu'}) I_R^2(z^{\mu'}) + O(\epsilon^4)\end{aligned}$$

$$I_R(z^\mu) \equiv \int d\tau'\, G_R(z^\mu, z^{\mu'})$$

  - *Regularization via dimensional regularization, implies* $G_{\rm ret} \to G_R$ (*at all orders*)

Third order self-force and field (2)

- Radiated field in Detweiler-Whiting decomposition

$$\psi_{\rm rad}(x) = \int d\tau'\, G_{\rm ret}(x, z^{\mu'}) \bigg\{ - m c_1 + m^2 c_1 c_2 I_R(z^\mu) - m^3 c_1 c_2^2 \int d\tau'\, G_R(z^\mu, z^{\mu'}) I_R(z^{\mu'}) - \frac{m^3 c_1^2 c_3}{2}\, I_R^2(z^\mu) + O(\epsilon^4) \bigg\}$$

$$\phi_{\rm rad}(x) = \phi[\psi_{\rm rad}(x)]$$

2nd order expression agrees with Rosenthal (2006) for appropriate parameter choices

CRG, CQG (2012b)

Third order self-force and field (3)

- 3rd order self-force in Hadamard decomposition

$$
\begin{aligned}
m_{\text{eff}}(\tau)a^{\mu} = P^{\mu\nu}\Big\{ & m^2 c_1^2\big[f_\nu(z^\mu) + I_\nu^{\text{tail}}(z^\mu)\big] - m^3 c_1^2 c_2\Big[2 f_\nu(z^\mu) I_{\text{tail}}(z^\mu) + I_\nu^{\text{tail}}(z^\mu) I_{\text{tail}}(z^\mu) \\
& + \lim_{\epsilon\to 0^+}\int_{-\infty}^{\tau-\epsilon} d\tau' \nabla_\nu D_{\text{ret}}(z^\mu, z^{\mu'}) I_{\text{tail}}(z^{\mu'})\Big] \\
& + m^4 c_1^2 c_2^2\Big[f_\nu(z^\mu) I_{\text{tail}}^2(z^\mu) - \frac{1}{2\pi} f_\nu(z^\mu)\frac{DI_{\text{tail}}(z^\mu)}{d\tau} - \frac{1}{4\pi} I_\nu^{\text{tail}}(z^\mu)\frac{DI_{\text{tail}}(z^\mu)}{d\tau} \\
& + I_\nu^{\text{tail}}(z^\mu)\lim_{\epsilon\to 0^+}\int_{-\infty}^{\tau-\epsilon} d\tau' D_{\text{ret}}(z^\mu, z^{\mu'}) I_{\text{tail}}(z^{\mu'}) \\
& + I_{\text{tail}}(z^\mu)\lim_{\epsilon\to 0^+}\int_{-\infty}^{\tau-\epsilon} d\tau' \nabla_\nu D_{\text{ret}}(z^\mu, z^{\mu'}) I_{\text{tail}}(z^{\mu'}) \\
& + 2 f_\nu(z^\mu)\lim_{\epsilon\to 0^+}\int_{-\infty}^{\tau-\epsilon} d\tau' D_{\text{ret}}(z^\mu, z^{\mu'}) I_{\text{tail}}(z^{\mu'}) \\
& - \frac{1}{4\pi}\lim_{\epsilon\to 0^+}\int_{-\infty}^{\tau-\epsilon} d\tau' \nabla_\nu D_{\text{ret}}(z^\mu, z^{\mu'})\frac{DI_{\text{tail}}(z^{\mu'})}{d\tau'} \\
& + \lim_{\epsilon\to 0^+}\int_{-\infty}^{\tau-\epsilon} d\tau' \nabla_\nu D_{\text{ret}}(z^\mu, z^{\mu'}) \times \lim_{\epsilon'\to 0^+}\int_{-\infty}^{\tau'-\epsilon'} d\tau'' D_{\text{ret}}(z^{\mu'}, z^{\mu''}) I_{\text{tail}}(z^{\mu''})\Big] \\
& + \frac{m^4 c_1^3 c_3}{2}\Big[2 f_\nu I_{\text{tail}}^2(z^\mu) + I_\nu^{\text{tail}}(z^\mu) I_{\text{tail}}^2(z^\mu) \\
& + \lim_{\epsilon\to 0^+}\int_{-\infty}^{\tau-\epsilon} \nabla_\nu D_{\text{ret}}(z^\mu, z^{\mu'}) I_{\text{tail}}^2(z^{\mu'})\Big] + O(\varepsilon^4)\Big\},
\end{aligned}
\tag{136}
$$

where $I_{\text{tail}}(z^\mu)$ and $I_\nu^{\text{tail}}(z^\mu)$ are the following tail integrals:

$$
I_{\text{tail}}(z^\mu) = \lim_{\epsilon\to 0^+}\int_{-\infty}^{\tau-\epsilon} d\tau' D_{\text{ret}}(z^\mu, z^{\mu'}) \tag{137}
$$

$$
I_\nu^{\text{tail}}(z^\mu) = \lim_{\epsilon\to 0^+}\int_{-\infty}^{\tau-\epsilon} d\tau' \nabla_\nu D_{\text{ret}}(z^\mu, z^{\mu'}), \tag{138}
$$

Nonperturbative self-force effects

CRG (in preparation)

Assumptions

- Let only c_1 and c_2 be non-zero; all other coefficients vanish

$$C\big(\psi(z^\mu)\big) = 1 + c_1\psi(z^\mu) + \frac{c_2}{2}\psi^2(z^\mu)$$

$$S[z^\mu, \psi] = -\frac{1}{2}\int_x \psi_{,\alpha}\psi^{,\alpha} - m\int d\tau\left(1 + c_1\psi(z) + \frac{1}{2}c_2\psi^2(z)\right)$$

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$$G_{\rm cons}(x, x') = \frac{G_{\rm ret}(x, x') + G_{\rm adv}(x, x')}{2}$$

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- Continue to ignore finite-size effects

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- Wave equation

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$$\psi_R(r_o) = -\frac{mc_1 I_R(r_o)}{1 + mc_2 I_R(r_o)} \qquad \partial_r\psi_R(r_o) = -\frac{mc_1 \partial_r I_R(r_o)}{1 + mc_2 I_R(r_o)}$$

Nonperturbative conservative quantities (1)

- The components of the worldline equations of motion for circular orbits yield non-perturbative expressions for:

- Effective potential

$$V_{\rm eff}(r_o) = f(r_o)\frac{1 + r_o \partial_r \ln C(\psi_R(r_o))}{1 - r_o \partial_r \ln f^{1/2}(r_o)} \qquad f(r_o) = 1 - \frac{2M}{r_o}$$

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- "Gauge invariant" redshift

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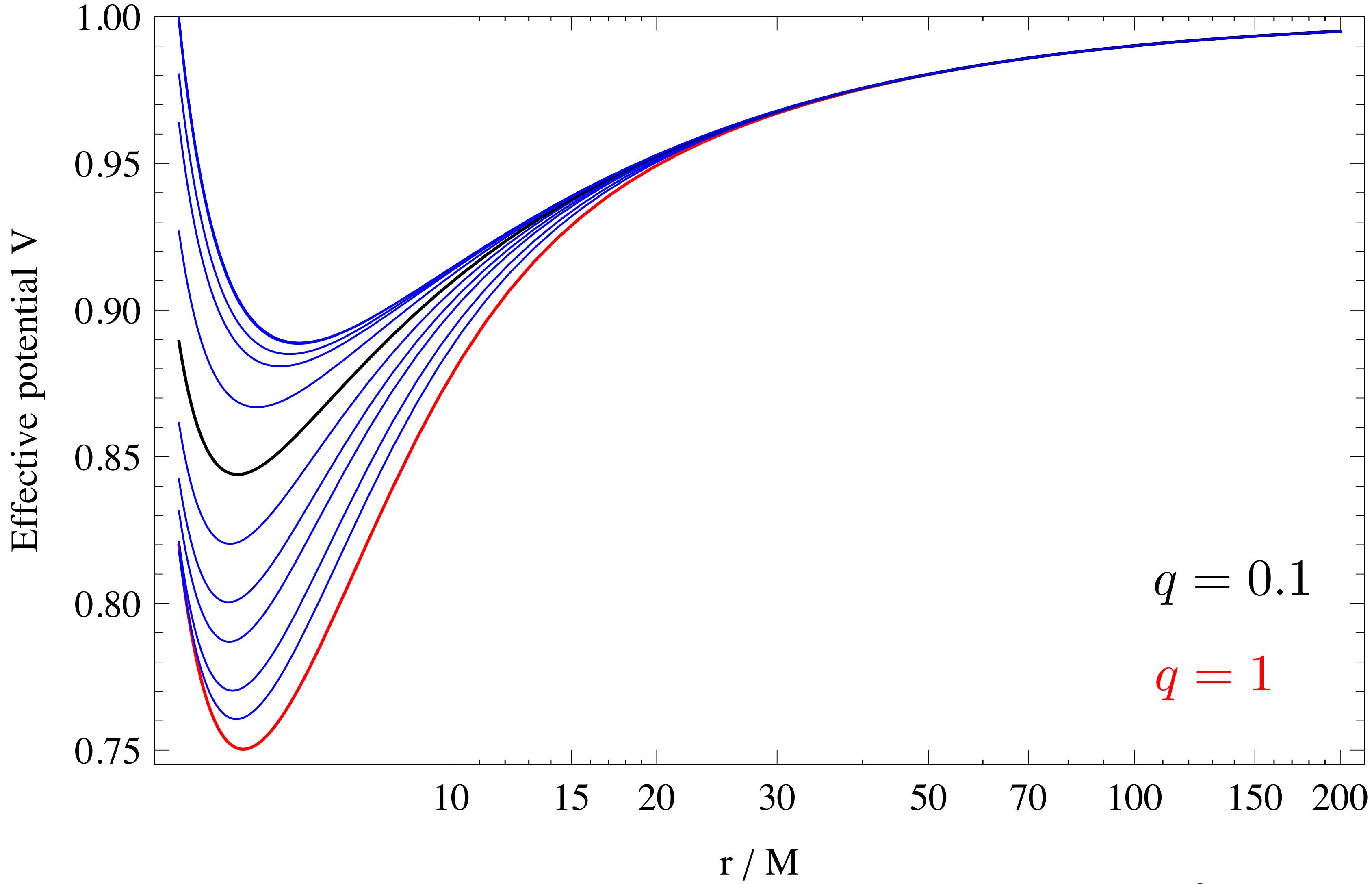
- Radial self-force

$$F^r_R(r_o) = -f(r_o)\partial_r \ln C(\psi_R(r_o))$$

Nonperturbative conservative quantities (2)

- Effective potential

$$V(r_o) = f(r_o)\frac{1 + r_o\partial_r \ln C(r_o)}{1 - r_o\partial_r \ln f^{1/2}(r_o)}$$



$c_1 = 2, \; c_2 = 0.1$

Nonperturbative conservative quantities (3)

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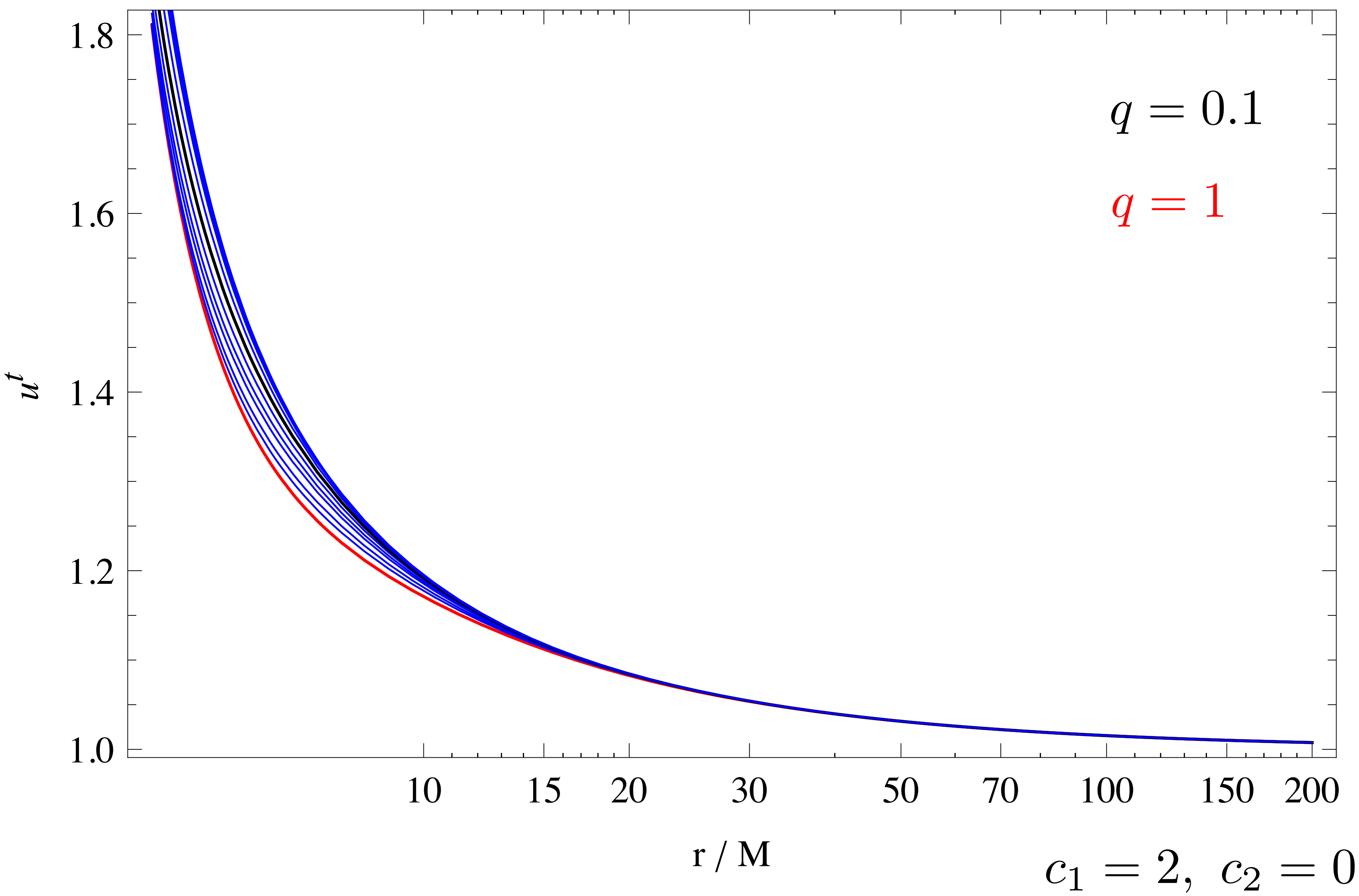

$c_1 = 2,\ c_2 = 0.1$

Improving perturbation theory

CRG (in progress)

An ambiguity at first order

- The effective action for 1st order self-force generates a 2nd order piece

$$ma^\mu = -mc_1(g^{\mu\nu} + u^\mu u^\nu)\nabla_\nu \psi_R(z) - mc_1 \psi_R(z) a^\mu$$

- One may either:

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- If keeping this term then

  - Can identify an effective mass for the particle

$$\big(m + mc_1 \psi_R(z)\big) a^\mu = -mc_1(g^{\mu\nu} + u^\mu u^\nu)\nabla_\nu \psi_R(z)$$

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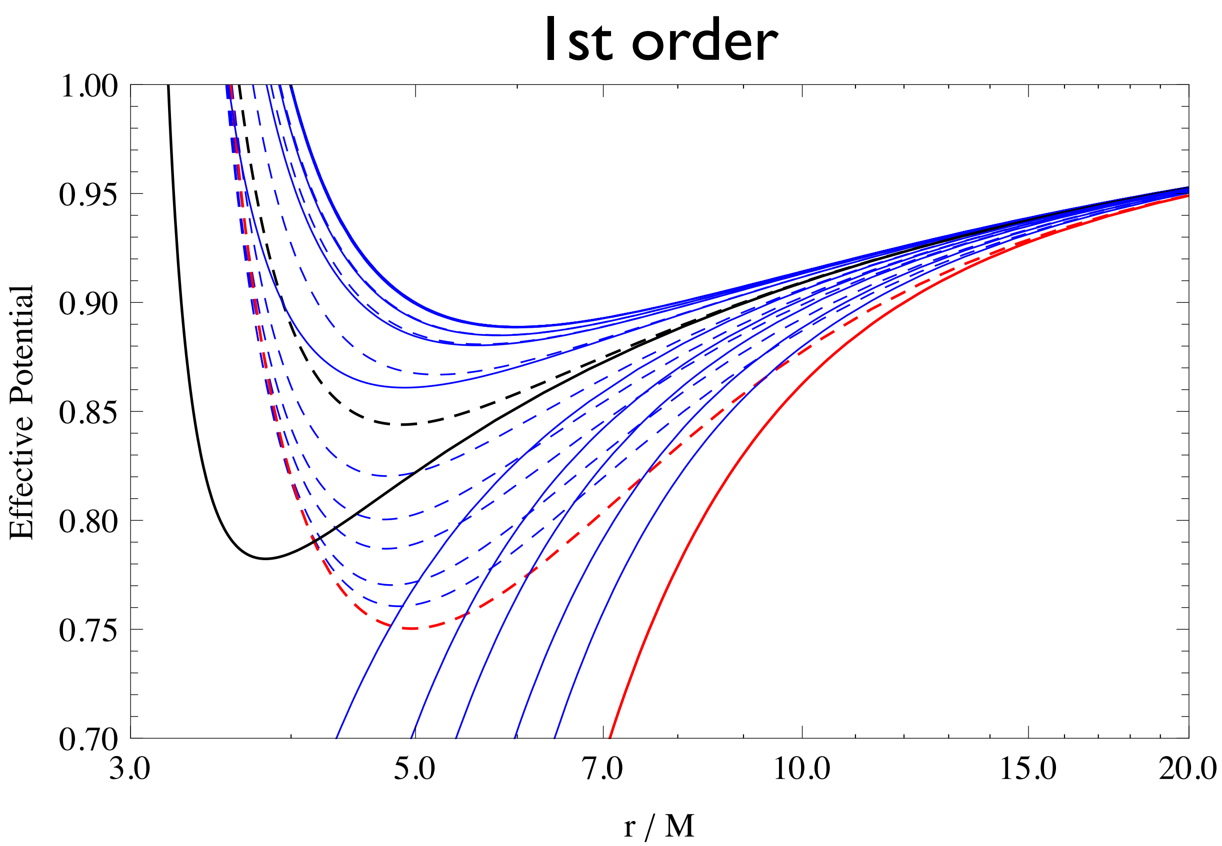
$$\big(m + mc_1\psi_R(z)\big)a^\mu = -mc_1(g^{\mu\nu} + u^\mu u^\nu)\nabla_\nu \psi_R(z)$$

- Can incorporate into a partially resummed expression of the 1st order self-force

$$ma^\mu = -\frac{mc_1(g^{\mu\nu} + u^\mu u^\nu)\nabla_\nu \psi_R(z)}{1 + c_1\psi_R(z)}$$

Improved 1st order self-force

Improved 1st order self-force effects



$$\Longleftarrow \quad V_{\mathrm{eff}}^{(1)} = V_{\mathrm{Schw}}\big(1 + c_1 r_o \partial_r \psi_R(r_o)\big)$$

Dashed lines are nonperturbative potential from earlier

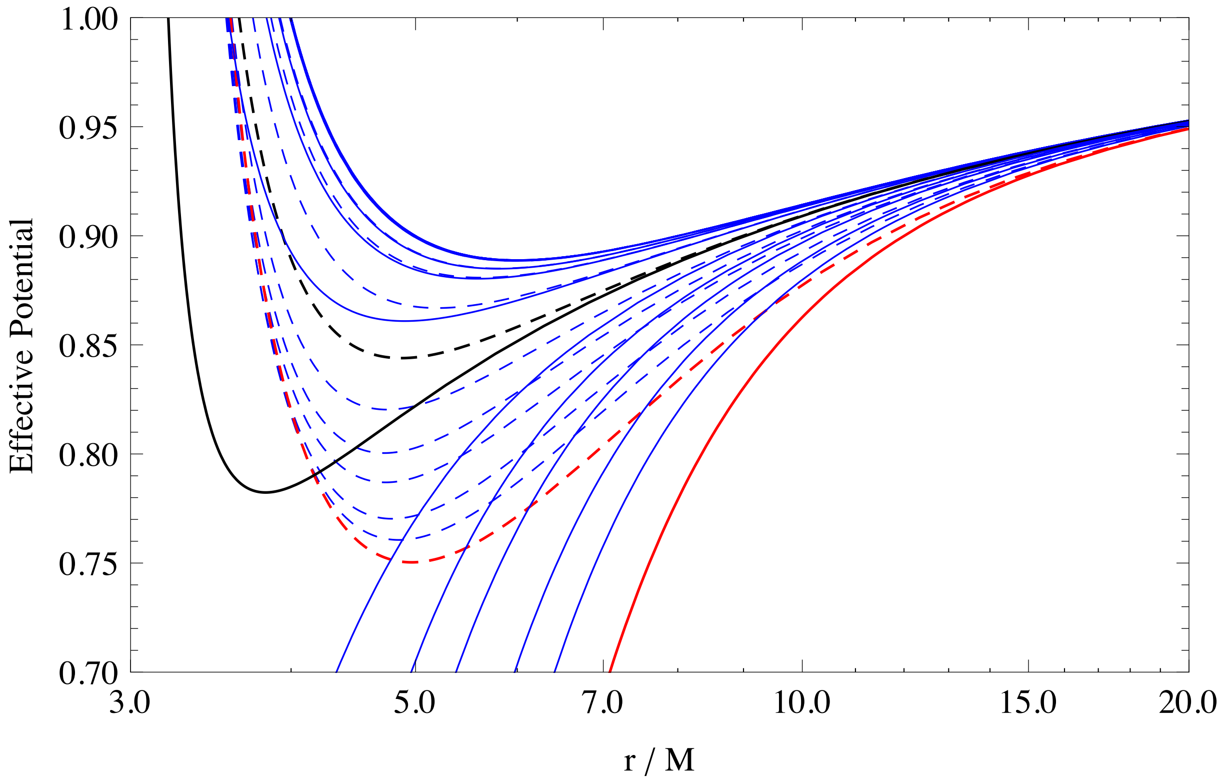
$$q = 0.1$$

$$q = 1$$

$$c_1 = 2,\ c_2 = 0.1$$

Improved 1st order self-force effects

1st order



$$V_{\text{eff}}^{(1)} = V_{\text{Schw}}\left(1 + c_1 r_o \partial_r \psi_R(r_o)\right)$$

$$V_{\text{eff}}^{\text{impr}} = V_{\text{Schw}}\left(\frac{1 + c_1\psi_R(r_o) + c_1 r_o \partial_r \psi_R(r_o)}{1 + c_1 \psi_R(r_o)}\right)$$

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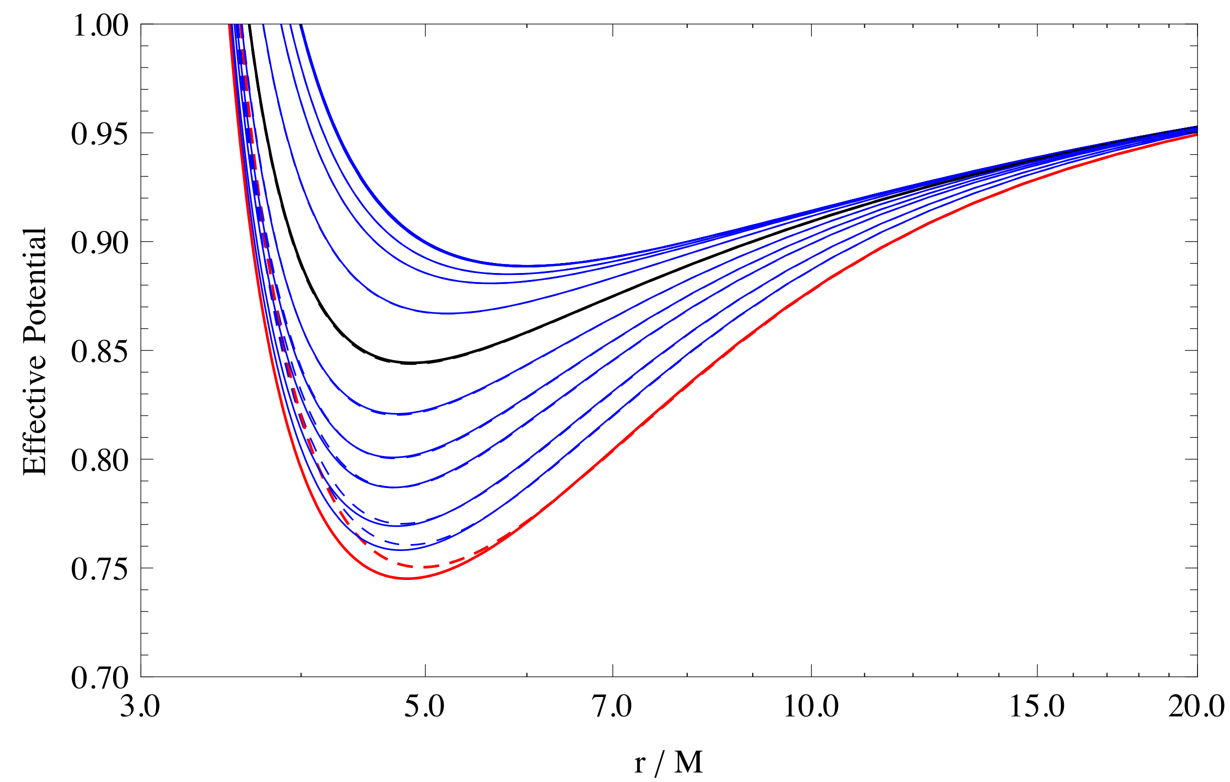

Dashed lines are nonperturbative potential from earlier

$q = 0.1$

$q = 1$

$c_1 = 2,\ c_2 = 0.1$

Improving first order gravitational self-force

- The effective action for 1st order self-force generates a 2nd order piece

$$ma^\mu = -mc_1(g^{\mu\nu} + u^\mu u^\nu)\nabla_\nu \psi_R(z) - mc_1 \psi_R(z) a^\mu$$

$$\Downarrow$$

$$-16\pi G m \left(\frac{1}{2} a^\mu u^\alpha u^\beta + (g^{\mu(\alpha} + u^\mu u^{(\alpha} a^{\beta)}) h^R_{\alpha\beta}(z) \right)$$

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- If keeping this term then can incorporate into a partially resummed expression of the 1st order self-force

$$F^r_R(r_o) = \frac{16\pi m P^{\mu\alpha\beta\nu} \nabla_\nu h^R_{\alpha\beta}(r_o)}{1 + 8\pi \big(h^R_{uu}(r_o) + g^{rr}(r_o) h^R_{rr}(r_o) \big)} + O(\epsilon^2)$$

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- ☐ A simple nonlinear scalar theory helps for gravitational self-force:

  - For understanding some conceptual aspects (e.g., regularization at higher orders)

  - For developing and studying algorithms for higher order computations

  - Provide a context to estimate errors of numerical self-force codes

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Conclusions

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- ☐ A simple nonlinear scalar theory helps for gravitational self-force:

  - *For understanding some conceptual aspects (e.g., regularization at higher orders)*

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- ☐ For a class of nonlinear scalar theories, the regular field can be resummed in the mass ratio for circular orbits

- ☐ A 2nd order piece comes from the 1st order action, which can be used to improve the first order perturbative expressions

  - *Worth studying for improving 1st order gravitational self-force results*

Extra slides

An ambiguity

- Worldline equations of motion

$$ma^\mu = -m(a^\mu + P^{\mu\nu}\nabla_\nu)C(z)$$

- Collect 4-acceleration to one side

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- Two equivalent interpretations:

1) Particle carries an effective mass

$$m_{\text{eff}} = mC(z)$$

2) Inertial mass but an effective self force

$$F^\mu_{\text{eff}}(\tau) = \frac{F^\mu_R(\tau)}{C(z)} = -mP^{\mu\nu}\nabla_\nu \ln C(z)$$

Comparison with Rosenthal's expression (I)

E. Rosenthal CQG (2006)

$$\Box\phi = \phi_{,\alpha}\phi^{,\alpha} - q\int d\tau\, \frac{\delta^4(x-z)}{g^{1/2}}$$

- Rosenthal developed a somewhat complicated procedure to derive the regular 2nd order perturbations

  - Investigate behavior of wave equation when $q \to 0$

  - Make ansatz for particular solution

  - Use physical considerations to identify divergent b.c.'s for field as field point approaches worldline

  - Solve the wave equation with those divergent b.c.'s

$$\phi(x) = q\int d\tau\, D_{\rm ret}(x, z^\mu) + \frac{q^2}{2}\left[\int d\tau\, D_{\rm ret}(x, z^\mu)\right]^2 - q^2\int d\tau\, D_{\rm ret}(x, z^\mu) I_R(z^\mu) + \cdots$$

Comparison with Rosenthal's expression (2)

$$\Box\phi = \phi_{,\alpha}\phi^{,\alpha} - q\int d\tau\,\frac{\delta^4(x-z)}{g^{1/2}}$$

- Rosenthal's model is a member of our class of nonlinear theories:

$$b_1 = -\frac{q}{m},\;\; b_2 = +2\frac{q}{m},\;\; a_3 = -6$$

- Psi field:

$$\psi_{\rm rad}(x) = q\int d\tau\, D_{\rm ret}(x, z^\mu) - q^2\int d\tau\, D_{\rm ret}(x, z^\mu) I_R(z^\mu) + \cdots$$

- Using the inverse of the field redefinition

$$\phi(x) = \psi(x) + \frac{1}{2}\psi^2(x) + \cdots$$

gives agreement with Rosenthal

$$\phi(x) = q\int d\tau\, D_{\rm ret}(x, z^\mu) + \frac{q^2}{2}\left[\int d\tau\, D_{\rm ret}(x, z^\mu)\right]^2 - q^2\int d\tau\, D_{\rm ret}(x, z^\mu) I_R(z^\mu) + \cdots$$

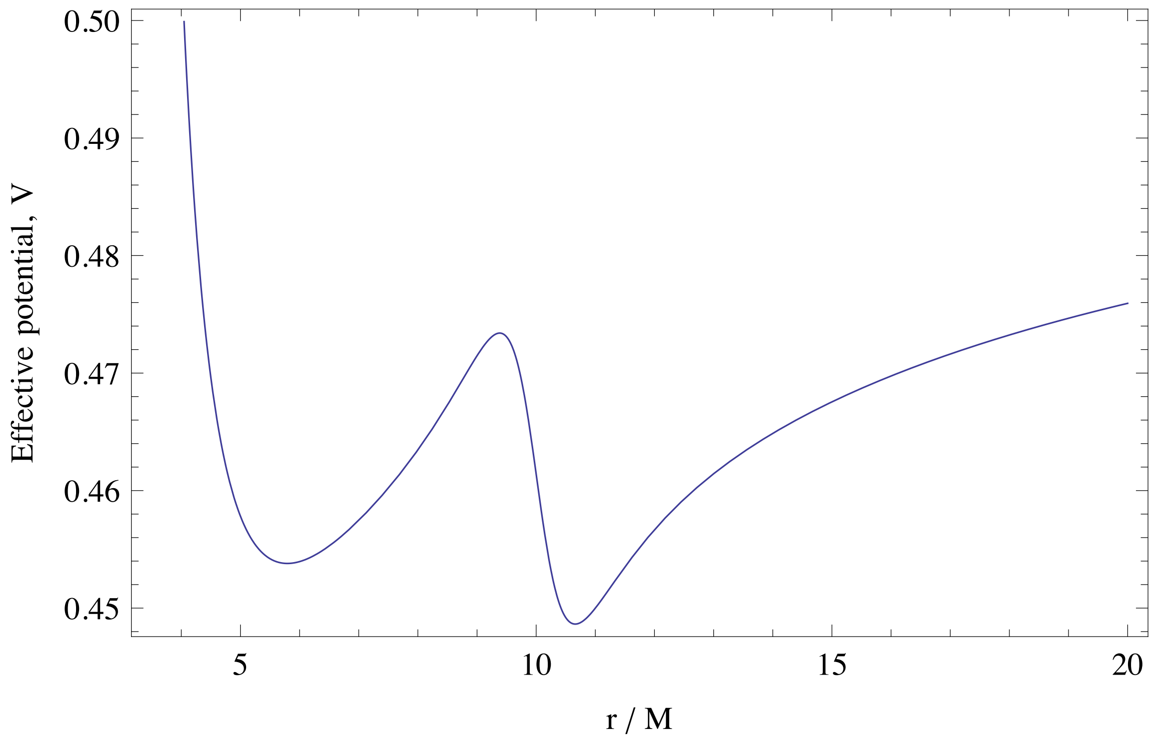
Scalar perturbations through 3rd order

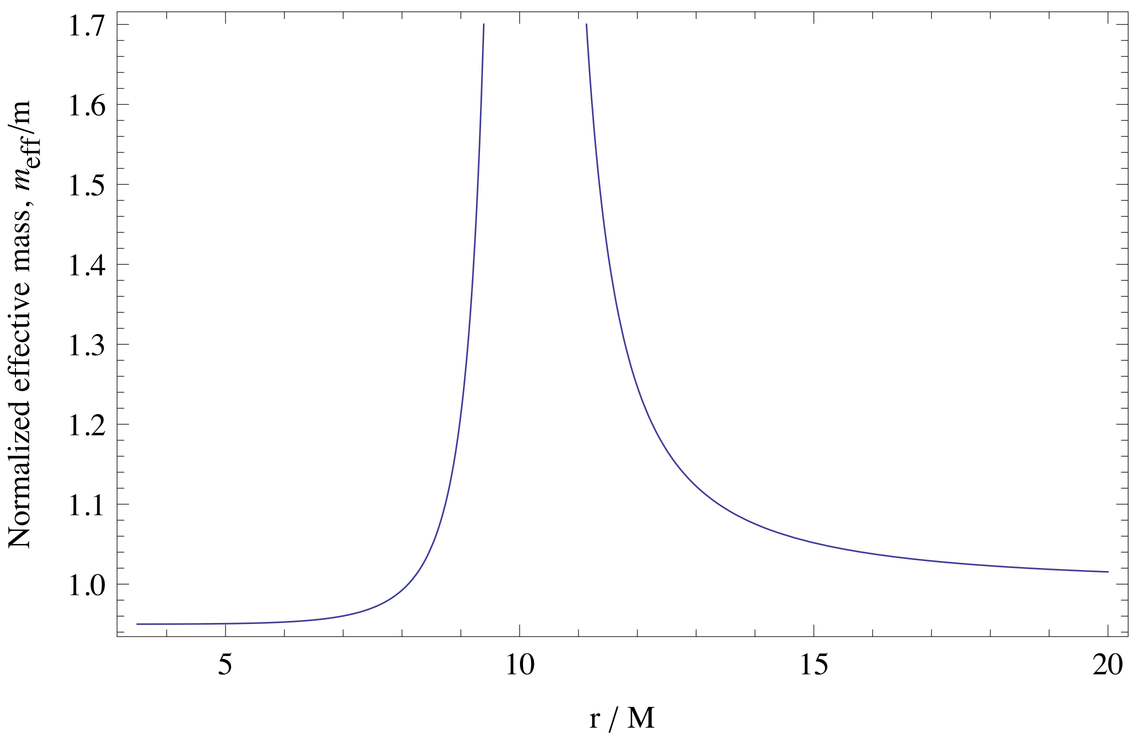
- Combining all the contributions to the scalar perturbations then gives the radiative field

$$\psi_{\rm rad}(x) = \int d\tau' \, D_{\rm ret}(x, z^{\mu'}) \Big\{ -mc_1 + m^2 c_1 c_2 I_R(z^{\mu'}) - \frac{m^3 c_1^2 c_3}{2} I_R^2(z^{\mu'}) - m^3 c_1 c_2^2 \int d\tau'' \, D_R(z^{\mu'}, z^{\mu''}) I_R(z^{\mu''}) + O(\varepsilon^4) \Big\}$$

Rich structures

- If the c_2 parameter is much larger than c_1 than one finds rich structure in the non-perturbative orbital quantities





$$q = 1$$

$$c_1 = 1,\ \ c_2 = 10$$

Summary

- We constructed a class of nonlinear scalar models analogous to the perturbative description of EMRIs in GR

- Calculated the scalar perturbations and SF through 3rd order

- Explicitly showed that DW scheme is valid at higher orders

- A subclass of these models can be resummed exactly to yield non-perturbative expressions in the mass ratio

- Showed how various orbital quantities vary with full mass ratio range

- Perturbative SF can be improved by retaining the "effective mass" piece