This test is:

- closed-book

- closed-notes

- no-calculator

- 75 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of this outcome should be clear.

There are four questions, some with multiple parts; each question is worth a total of 25 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

Do not turn this page until directed to begin

1. (25 pts total) This question concerns non-negative integers, where each digit can be one of the 10 options $0, 1, \ldots, 9$.

a.) (6 pts) How many different 4-digit numbers are there?

b.) (6 pts) How many different 4-digit numbers without any repeated digits are there?

c.) (6 pts) How many 5-digit numbers either begin with 3, or end with 5?

d.) (7 pts) How many different 7-digit numbers can be formed from a permutation of the digits in the number 3,453,433?

2. (25 pts total) An n-bit string is a sequence of n bits, where each bit is either 0 or 1.

a.) (5 pts) How many 8-bit strings either begin with 010, or begin with 111?

b.) (5 pts) How many 8-bit strings either begin with 010, or end with 000?

c.) (5 pts) How many 8-bit strings either begin with 01111, or end with 10000?

d.) (10 pts) A total of 60 students are asked which types of courses they are enrolled in for next semester.

30 are enrolled in mathematics courses
16 are enrolled in english courses
36 are enrolled in philosophy courses
12 are enrolled in both mathematics and english courses
17 are enrolled in both english and philosophy courses
23 are enrolled in both philosophy and mathematics courses
5 are enrolled in all three kinds of courses

How many students are enrolled in none of these types of courses?

3. (25 pts total)

a.) (6 pts) 34 pencils are distributed among 6 people. What is the largest number of pencils that you can be sure at least one person possesses?

b.) (6 pts) An editor randomly chooses n words from an English dictionary. How large must n be to ensure that at least 5 of the chosen words begin with the same letter?

c.) (6 pts) A bowl contains a number of red, green, and blue marbles. How many marbles must the bowl contain to ensure that there are either at least 2 red marbles, at least 6 green marbles, or at least 12 blue marbles?

d.) (7 pts) A box contains 23 books, and you are told that there are either at least 5 English books, at least 8 French books, at least 4 German books, or at least k Latin books, for some $k \in \mathbb{N}$. What is the largest possible value of k?

4. (25 pts total)

a.) (5 pts) How many subsets of the set $\{1, 3, a, 7, b, 4, c, q\}$ contain exactly 4 elements?

b.) (5 pts) How many different 4-person committees can be formed from a group of 12 people?

c.) (5 pts) A 25-person company must simultaneously allocate 12 employees to attend a conference, 6 employees to visit a client, and 7 employees to stay at the company office. How many different ways of allocating employees are possible?

d.) (5 pts) Determine the number of distinct permutations (with repetition) of the letters in the word ALFALFA.

e.) (5 pts) A pack of cards has four suits (classifications): spades, clubs, diamonds, and hearts. If 5 cards are dealt to you at random, how many distinct assortments of suits are possible? (E.g., "3 clubs, 2 hearts", and "5 spades" are two possible such assortments.)