Exam 2 **Name:** ______________________________

MTH182, Section 03 March 12, 2015

This test is:

- closed-book
- closed-notes
- no-calculator
- 75 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of this outcome should be clear.

There are four questions, some with multiple parts; each question is worth a total of 25 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

Do not turn this page until directed to begin

1. (25 pts total) This problem is concerned with a function f defined as follows: let $f : A \to B$, with $A = \{a, b, c, d, e\}$ and $B = \{x, y, z\}$, be the function

$$f = \{(a, x),\ (b, x),\ (c, z),\ (d, x),\ (e, z)\}$$

a.) (5 pts) Determine the domain, codomain, and range of f.

b.) (5 pts) Determine the image of d.

c.) (5 pts) Determine whether y is an image.

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d.) (5 pts) Determine $f(X)$ where $X = \{a, c, d\}$.

e.) (5 pts) Give any example of a new function g whose domain is $B = \{x, y, z\}$ and codomain is $A = \{a, b, c, d, e\}$.

2. (25 pts total) This question is concerned with the sets A and B defined as

$$A = \{1, 2, 3, 4\}, \qquad B = \{a, b, c, d, e\}$$

a.) (6 pts) Write down any function function $f : A \to B$.

b.) (6 pts) Write down any function function $g : A \to B$ that is injective ("one-to-one") but *not* surjective ("onto").

$$A = \{1, 2, 3, 4\}, \qquad\qquad B = \{a, b, c, d, e\}$$

c.) (6 pts) Write down any function function $h : A \to B$ that is *neither* injective *nor* surjective.

d.) (7 pts) Define your own *nonempty* sets C and D, and a function $g : C \to D$ such that g is *not* injective but is surjective.

3. (25 pts total) This question concerns bijections and cardinality.

a.) (6 pts) A function $f : \mathbb{Z} \to \mathbb{Z}$ is defined by $f(n) = 5n + 1$. Determine, with justification, whether f is bijective.

b.) (6 pts) Let $A = \{a, b, c\}$ and $B = \{w, x, y, z\}$, with $f : A \to B$ defined by $f(a) = y$, $f(b) = z$, and $f(c) = w$. Determine, with explanation, whether f has an inverse function from B to A.

c.) (13 pts) Show that the cardinality of $\mathbb{Z}$ equals the cardinality of $3\mathbb{Z}$, where $3\mathbb{Z} = \{3k \mid k \in \mathbb{Z}\}$.

4. (25 pts total) This question concerns growth of functions.

a.) (12 pts total) Let $f, g : \mathbb{N} \to [0, \infty)$. Define $f(n) = 2n^2$ and $g(n) = 5n$ for all $n \in \mathbb{N}$. Show that g is big O of f, i.e., $g = O(f)$.

b.) (13 pts total) Let $f, g : \mathbb{N} \to [0, \infty)$. Define $f(n) = 3n^3 + n^2$ and $g(n) = n^3$ for all $n \in \mathbb{N}$. Show that f is big theta of g, i.e., $f = \Theta(g)$.