Mathematics Department, University of Massachusetts Dartmouth

Discrete Mathemtics II

MTH182 – Section 03 – Spring 2015

Review for Exam 2

Exam 2 is Thursday, March 12 from 8:00-9:15am

Reading: Discrete Mathematics, first edition, section Sections 5.3-5.5, 6.1-6.2
Problems sets 5 and 6
Chapter 5 supplementary exercises: 17, 19, 23, 29, 31, 37, 39, 41
Chapter 6 supplementary exercises: 5, 7, 9

Summary of concepts

The following is a list of the major topics covered in this exam:

- Functions
- Injections, surjections, bijections
- Composition of functions, inverse functions
- Cardinality
- Finite, denumerable, countable, uncountable sets
- Algorithms
- Algorithmic complexity: "big-O" and "big-theta" relations and growth

What follows are a list of *suggested* exercises that will help you review for the exam. It is *not* a replacement for the week-by-week problem sets.

Chapter 5 Supplementary Exercises

17. Let $A = \{0, 1, 3, 4\}$ and $B = \{-2, -1, 0, 1, 2\}$.
(**a**) Given an example of a function $f : A \to B$.
(**b**) What is the range of f?

19. Let $A = \{1, 3, 4\}$ and $B = \{-2, -1, 0, 1, 4\}$. For $a \in A$, define $f(a) = b$, where $|b| = a$. Is f a function from A to B?

23. Define $f : \mathbb{N} \to \mathbb{N}$ by $f(n) = \lceil n/3 \rceil$.
(**a**) Is f one-to-one?
(**b**) Is f onto?

29. Show that there is a function $f : \mathbb{R} \to \mathbb{R}$ that is
(**a**) onto but not one-to-one.
(**b**) one-to-one but not onto.

31. Show that the functio $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = ax + b$, where $a, b \in \mathbb{R}$ and $a \neq 0$ is bijective.

37. Show that the set A of negative integers if denumerable.

39. Prove that if A and B are disjoint countable sets, then $A \cup B$ is countable.

41. Determine, with explanation, whether the following is true or false. If A and B are denumerable sets, then $A \cap B$ is denumerable.

Chapter 6 Supplementary Exercises

5. Show that $n \log n = O(n^2)$.

7. For the function f defined by $f(n) = \frac{2n^3+n}{n+2}$ for each $n \in \mathbb{N}$, show that $f(n) = O(n^2)$.

9. Let f and g be the functions defined by $f(n) = n^2 + 3n + 1$ and $g(n) = n^3$ for each $n \in \mathbb{N}$. Show that $f(n) = O(g(n))$, but $g(n) \neq O(f(n))$.