Exam 1 **Name:** ______________________________

MTH182, Section 03 February 19, 2015

This test is:

- closed-book
- closed-notes
- no-calculator
- 75 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of this outcome should be clear.

There are four questions, some with multiple parts; each question is worth a total of 25 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

DO NOT TURN THIS PAGE UNTIL DIRECTED TO BEGIN

1. (25 pts total) **Use mathematical induction** to prove that, for all $n \in \mathbb{N}$,

$$\sum_{k=1}^{n}(4k-3) = n(2n-1)$$

2. (25 pts total) The following question concern a relation R on a set S.

a.) (5 pts) *Define* what it means for R to be reflexive.

b.) (5 pts) *Define* what it means for R to be symmetric.

c.) (10 pts) On the set $S = \{1, 2, 3, 4, 5\}$, let R be defined by

$$R = \{(1,1), (1,3), (1,4), (1,5), (2,2), (3,1), (3,3), (4,2), (4,4), (5,1), (5,5)\}$$

Determine whether or not R is reflexive, symmetric, and/or transitive. *Justify your answer.*

d.) (5 pts) On the set $S = \{1, 2, 3\}$, give an example of a relation R that is <u>both</u> reflexive and symmetric, but is <u>not</u> transitive.

3. (25 pts total) This question concerns equivalence relations on a set S.

a.) (5 pts) Let $S = \{1, 2, 3, 4\}$, with R defined on S as follows: $a\,R\,b$ if $a + b$ is even. Identify the equivalence classes of S induced by R.

b.) (10 pts) Let $S = \{1, 2, 3, 4, 5, 6\}$. Suppose R is an equivalence relation on S with equivalence classes $\{1, 2\}$, $\{3, 4\}$, $\{5, 6\}$. What is R?
(I am asking, for example, for you to write out R as a set of ordered pairs from S.)

c.) (10 pts) Let $S = \mathbb{Z}$ be the set of all integers. Define R on S as follows: $a\,R\,b$ if either $a + b = 0$ or if $a - b = 0$. Describe the distinct equivalence classes of S induced by R.

4. (25 pts total) Let $S = \{2, 4, 6, 8, \ldots\}$. (I.e., S is the set of all positive even numbers.) Define R as a relation on S as follows: for any elements a and b in S, a is related to b if $a + b$ is a multiple of 4, i.e., if $a + b = 4k$ for some $k \in \mathbb{Z}$. Prove that R is an equivalence relation.