Mathematics Department, University of Massachusetts Dartmouth
Discrete Mathemtics II
MTH182 – Section 03 – Spring 2015
Review for Exam 1
Exam 1 is Thursday, Feb 19 from 8:00-9:15am

Reading:	Discrete Mathematics, first edition, section Sections 4.1-4.4, 5.1, 5.2
	Chapter 4 supplementary exercises: 1, 3, 9, 11, 17, 19, 21
	Chapter 5 supplementary exercises: 1, 3, 5, 7, 11, 13

Summary of concepts

The following is a list of the major topics covered in this exam:

- Proof by mathematical induction
- Sequences
- Proof by the Strong Principle of Mathematical Induction
- Relations, product spaces
- Relation properties: symmetry, reflexivity, transitivity
- Equivalence relations, equivalence classes, partitions

What follows are a list of *suggested* exercises that will help you review for the exam. It is *not* a replacement for the week-by-week problem sets.

Chapter 4 Supplementary Exercises

1. Prove that $1 + 3 + 6 + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{6}$ for every positive integer n.
3. Prove that $1^2 + 3^2 + \cdots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$ for every positive integer n.
9. Prove that $2^n > n^3$ for every integer $n \geq 10$.
11. Prove by induction that $n^n > n!$ for every integer $n \geq 2$.
17. Let a and b be two real numbers. Use induction to prove that $\sum_{i=0}^{n}(a+ib) = \frac{1}{2}(n+1)(2a+nb)$.
19. A sequence $a_0, a_1, a_2, \ldots,$ of integers is defined recursively by $a_0 = 0$ and $a_n = 3a_{n-1}+3^{n-1}$.
21. A sequence $\{a_n\}$ is defined recursively by $a_1 = 2$, $a_2 = 6$, and $a_n = 2a_{n-1} - a_{n-2} + 2$ for $n \geq 3$. Prove that $a_n = 2^n$ for every positive integer n.

Chapter 5 Supplementary Exercises

1. For an integer $n \geq 2$, let $S_n = \{1, 2, \ldots, n\}$, and let T_n be the set of all 2-element subsets of S_n. For $A, B \in T_n$, define the relation R_n on T_n by $A \; R_n \; B$ if $A \cap B = \varnothing$. For $n = 4$, list all elements of R_n.

3. Let $A = \{1, 2, 3, 4\}$. For the relation $R = \{(1, 1), (2, 2), (2, 3), (3, 2), (4, 4)\}$ on A, determine which of the properties reflexive, symmetric, transitive R possesses.

5. Let R be the relation on defined on $\mathbb{Z}$ by $a\ R\ b$ if $a = b$ or $a = 2b$.

(**a**) Given an example of two integers that are related by R, and two integers that are not.

(**b**) Which of the properties reflexive, symmetric, transitive does R possess?

7. A relation R is defined on $\mathbb{R}$ by $a\ R\ b$ if $ab \leq 0$.

(**a**) Given an example of two real numbers that are related by R, and two real numbers that are not.

(**b**) Which of the properties reflexive, symmetric, transitive does R possess?

11. A relation N on a nonempty set A is defined to be **circular** if whenever $a\ R\ b$ and $b\ R\ c$, then $c\ R\ a$ for all $a, b, c \in A$. Prove that a relation R on A is an equivalence relation if and only if R is reflexive and circular.

13. A relation R is defined on $\mathbb{N}$ by $a\ R\ b$ if $a^2 + b^2$ is even.

(**a**) Show that R is an equivalence relation.

(**b**) Describe the distinct equivalence classes resulting from R.