Midterm 1
MTH182, Section 03

Name: ____________________

February 15, 2013

This test is:

- closed-book
- closed-notes
- no-calculator
- 50 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of this outcome should be clear.

There are four questions with multiple parts; each question is worth a total of 25 points. There is one extra credit questions worth 10 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

Do not turn this page until directed to begin

1. (25 points total) Complete the following exercises.

a.) (5 pts) Find an explicit formula for the sequence a_n when

$$a_1 = 1, \quad a_2 = -3, \quad a_3 = 5, \quad a_4 = -7, \quad a_5 = 9, \ldots$$

b.) (5 pts) Compute $\sum_{n=-1}^{2} (n^2 - 3n)$

c.) (5 pts) Compute $\prod_{q=3}^{6} (q - 2)$

d.) (5 pts) Let n be an integer greater than 2. Write the following without using factorial notation:

$$\frac{(n!)^2}{(n-2)!(n+3)!}$$

e.) (5 pts) Compute the following:

$$\binom{12}{10} - \binom{8}{4}$$

2. (25 points total) Consider the formula:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \qquad\qquad n \geq 1.$$

a.) (17 pts) Prove the above formula using induction.

b.) (8 pts) Show using part a that

$$\sum_{k=1}^{n} k^2 = \binom{n+2}{3} + \binom{n+1}{3}.$$

(This is not an induction problem.)

3. (25 points total) This problem concerns basic set notation and theory.

a.) (5 pts) Write down the elements of $A \times B$, where $A = \{x, y\}$ and $B = \{3, 5\}$.

b.) (5 pts) Write down the elements of $\mathcal{P}(A)$ (the power set of A), with $A = \{x, y\}$.

c.) (5 pts) If $A = \{x, y, 1, 3, z\}$ and $B = \{3, 5, x, 4, 2\}$, compute $A - B$.

d.) (5 pts) For (a, b) an interval on the real line, let $A_k = \left[0, \frac{1}{k}\right)$. Compute $\bigcap_{k=1}^{15} A_k$.

e.) (5 pts) Let $A = \{x, y\}$. Compute $A \times \mathcal{P}(A)$.

4. (25 points total) The following problem concerns proving operations on sets. In all the following, assume some ambient space of elements U.

a.) (10 pts total) Use an element argument to prove that $(A \cup B)^c = A^c \cap B^c$. I.e., do not cite the set-valued versions of DeMorgan's Laws.

b.) (5 pts) Provide a counterexample to the statement $A \subseteq \mathcal{P}(A \times A)$.

c.) (10 pts) Prove the following if it is true, or find a counterexample if it is false: For sets A and B,

$$A = (A \cup B) - (B - A).$$