Mathematics Department, University of Massachusetts Dartmouth

Discrete Mathemtics II

MTH182 – Section 03 – Spring 2015

Problem set 8

Division and congruence

Reading:	Discrete Mathematics, first edition, section Sections 7.3, 7.4
	Section 7.3: 1, 3, 5, 9, 11, 17, 23
	Section 7.4: 1, 3, 5, 7, 9, 13, 15, 17

Section 7.3

1. For each of the following pairs of integers m, n of integers, find the quotient q and remainder r when m is divided by n. Then write $m = nq + r$.

(**a**) $m = 48$, $n = 11$.

(**b**) $m = 0$, $n = 11$.

(**c**) $m = -48$, $n = 11$.

(**d**) $m = 9$, $n = 11$.

3. Determine $(m \operatorname{div} n)$ and $(m \bmod n)$ for the following pairs m, n of integers.

(**a**) $m = 47$, $n = 14$

(**b**) $m = 81$, $n = 22$

(**c**) $m = 180$, $n = 45$

(**d**) $m = 24$, $n = 25$

(**e**) $m = 0$, $n = 15$

(**f**) $m = -55$, $n = 27$

5. Show that if n is an odd integer, then n^2 has a remainder of 1 when divided by 4.

9. For each integer n, prove that

(**a**) 3 divides one of the integers n, $n + 1$, and $2n + 1$

(**b**) 3 divides one of the integers n, $2n - 1$, and $2n + 1$

11. Let a, b, c, d, and e be consecutive integers. Use the Division Algorithm to prove that 5 divides one of these five integers.

17. Let $a, b \in \mathbb{Z}$. Prove that if $3 \nmid a$ and $3 \nmid b$, then $3|(a^2 - b^2)$.

23. Let $n \in \mathbb{Z}$. Prove that if $3|(n^2 + 1)$, then $3 \nmid (2n^2 + 1)$.

Section 7.4

1. For the following integers a, b, and n, determine whether a is congruent to b modulo n. If so, write $a \equiv b (\bmod n)$; if not, write $a \not\equiv b (\bmod n)$.

(**a**) $a = 47$, $b = 23$, $n = 8$
(**b**) $a = 18$, $b = 38$, $n = 5$
(**c**) $a = 20$, $b = 10$, $n = 3$
(**d**) $a = 12$, $b = 12$, $n = 13$
(**e**) $a = 37$, $b = 35$, $n = 2$

3. Given an example of four integers that are congruent to 4 modulo 13.

5. Recall Theorem 7.25 in the book: *Let a, b, and $n \geq 2$ be integers. Then $a \equiv b (\bmod n)$ if and only if $a = b + kn$ for some integer k.*
Prove the following:
Let a, b, and $n \geq 2$ be integers. Prove that $a \equiv b (\bmod n)$ if and only if $b = a + \ell n$ fo some integer ℓ.

7. Let a, b, m, and n be integers with m, $n \geq 2$ and $m|n$. Show that if $a \equiv b (\bmod n)$ then $a \equiv b (modm)$.

9. Let a, b, c, d, and $n \geq 2$ be integers. Show that if $a \equiv b (\bmod n)$ and $c \equiv d (\bmod n)$, then $a - c \equiv b - d (\bmod n)$.

13. Let $a, b \in \mathbb{Z}$. Prove that if $a \equiv 0 (\bmod 5)$ and $b \equiv 2 (\bmod 5)$, then $a^2 + b^2 = 4 (\bmod 5)$.

15. Let $a, b \in \mathbb{Z}$. Prove that if $a^2 + 2b^2 \equiv 0 (\bmod 3)$, then either both a and b are congruent to 0 modulo 3, or neither a nor b is congruent to 0 modulo 3.

17. For each of the following pairs a, b or integers and an integer $n \geq 2$, compute $a \bmod n$ and $b \bmod n$ and use this to determine whether $a \equiv b (\bmod n)$.

(**a**) $a = 47$, $b = 23$, $n = 8$
(**b**) $a = 31$, $b = 43$, $n = 6$
(**c**) $a = 27$, $b = -15$, $n = 7$
(**d**) $a = 35$, $b = -11$, $n = 12$
(**e**) $a = 0$, $b = -2$, $n = 2$
(**f**) $a = 43$, $b = 29$, $n = 9$