MATHEMATICS DEPARTMENT, UNIVERSITY OF MASSACHUSETTS DARTMOUTH

Discrete Mathemtics II

MTH182 – Section 03 – Spring 2015

Problem set 7

Divisibility and Primes

Reading:	Discrete Mathematics, first edition, section Sections 7.1, 7.2
	Section 7.1: 1, 3, 5, 7, 11
	Section 7.2: 1, 7, 11, 13

Section 7.1

1. For each pair a, b of integers, determine whether $a|b$. If $a|b$, then find an integer c such that $b = ac$.
   (**a**) $a = 7$ and $b = -70$
   (**b**) $a = 16$ and $b = -40$
   (**c**) $a = 1$ and $b = 10$
   (**d**) $a = 8$ and $b = -8$
   (**e**) $a = 14$ and $b = 0$
   (**f**) $a = 0$ and $b = 14$

3. Let a and b be integers with $a \neq 0$. Prove that if $a|b$, then $a|(-b)$ and $(-a)|b$.

5. Let a, b, and c be integers with $a \neq 0$ and $c \neq 0$. Prove that $ac|bc$ if and only if $a|b$.

7. Disprove the following: Let a and b be integers with $a \neq 0$ and $b \neq 0$. If $a|b$ and $b|a$, then $a = b$.

11. Prove that $3|(4n^3 + 5n)$ for every nonnegative integer n.

Section 7.2

1. Express each of the following integers as a product of primes.
   (**a**) 250
   (**b**) 297
   (**c**) 2662
   (**d**) 1225
   (**e**) 891

7. Of course, 11 is a prime.
   (**a**) Show that 111 is not a prime.
   (**b**) Show that 1111 is not a prime.
   (**c**) Show that $111,111$ is not a prime.
   (**d**) Is $11,111$ a prime?

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11. Show that only one prime can be expressed as $n^3 + 1$ for some positive integer n.

13. Goldbach's Conjecture states that every even integer $n \geq 4$ can be expressed as the sum of two primes. Goldbach also conjectured that every integer $n \geq 3$ can be written as the sum of three integers, each of which is either 1 or a prime. Prove that if Goldbach's conjecture is true, then this conjecture is also true.