Section 7.1

1. For each pair \(a, b\) of integers, determine whether \(a|b\). If \(a|b\), then find an integer \(c\) such that \(b = ac\).
   (a) \(a = 7\) and \(b = -70\)
   (b) \(a = 16\) and \(b = -40\)
   (c) \(a = 1\) and \(b = 10\)
   (d) \(a = 8\) and \(b = -8\)
   (e) \(a = 14\) and \(b = 0\)
   (f) \(a = 0\) and \(b = 14\)

3. Let \(a\) and \(b\) be integers with \(a \neq 0\). Prove that if \(a|b\), then \(a|(-b)\) and \((-a)|b\).

5. Let \(a, b,\) and \(c\) be integers with \(a \neq 0\) and \(c \neq 0\). Prove that \(ac|bc\) if and only if \(a|b\).

7. Disprove the following: Let \(a\) and \(b\) be integers with \(a \neq 0\) and \(b \neq 0\). If \(a|b\) and \(b|a\), then \(a = b\).

11. Prove that \(3|(4n^3 + 5n)\) for every nonnegative integer \(n\).

Section 7.2

1. Express each of the following integers as a product of primes.
   (a) 250
   (b) 297
   (c) 2662
   (d) 1225
   (e) 891

7. Of course, 11 is a prime.
   (a) Show that 111 is not a prime.
   (b) Show that 1111 is not a prime.
   (c) Show that 111,111 is not a prime.
   (d) Is 11,111 a prime?
11. Show that only one prime can be expressed as \( n^3 + 1 \) for some positive integer \( n \).

13. Goldbach’s Conjecture states that every even integer \( n \geq 4 \) can be expressed as the sum of two primes. Goldbach also conjectured that every integer \( n \geq 3 \) can be written as the sum of three integers, each of which is either 1 or a prime. Prove that if Goldbach’s conjecture is true, then this conjecture is also true.