Mathematics Department, University of Massachusetts Dartmouth

Discrete Mathemtics II

MTH182 – Section 03 – Spring 2015

Problem set 6

Algorithms and function growth

Reading:	Discrete Mathematics, first edition, section Sections 6.1, 6.2
	Section 6.1: 9, 11
	Section 6.2: 1, 3, 7, 9, 11, 15

Section 6.1

9. Illustrate algorithm 6.8 in the book for $k = 11$ and s: 9, 10, 14, 11.

11. Write an algorithm that determines whether a sequence s: a_1, a_2, $\ldots$, a_n of n numbers contains any negative numbers.

Section 6.2

1. For function $f : \mathbb{N} \to \mathbb{R}^+$ and $g : \mathbb{N} \to \mathbb{R}^+$, $f = O(g)$ if there is a positive constant C and a positive integer k such that $f(n) \leq Cg(n)$ for every integer $n \geq k$. Show that there is a positive constant C' such that $f(n) \leq C'g(n)$ for every positive integer n.

3. Let $f : \mathbb{N} \to \mathbb{R}^+$ and $g : \mathbb{N} \to \mathbb{R}^+$ be functions defined by $f(n) = 5n + 7$ and $g(n) = n^2$ for all $n \in \mathbb{N}$. Show that $f = O(g)$ but $g \neq O(f)$.

7. For which of the following is $f(n) = O(n^2)$?

- **(a)** $f(n) = 2n + 5$
- **(b)** $f(n) = \lfloor n/2 \rfloor$
- **(c)** $f(n) = n^2 + 3n + 2$
- **(d)** $f(n) = n \log n$
- **(e)** $f(n) = n^2 \log n$
- **(f)** $f(n) = 2^n$

9. Let $f : \mathbb{N} \to \mathbb{R}^+$ and $g : \mathbb{N} \to \mathbb{R}^+$ be two functions defined by $f(n) = 2n + 1$ and $g(n) = n$ for all $n \in \mathbb{N}$. Show that $f = \Theta(g)$.

11. Let $f : \mathbb{N} \to \mathbb{R}^+$ and $g : \mathbb{N} \to \mathbb{R}^+$ be functions defined by $f(n) = n^2 + 4n + 1$ and $g(n) = n^2 + 4$ for all $n \in \mathbb{N}$. Show that $f = \Theta(g)$.

15. Let f and g be two functions defined by $f(n) = \frac{1}{2}n^2 + 5n + 1$ and $g(n) = 2n^2 + 3$. Show that $f(n) = \Theta(g(n))$.