

**Discrete Mathematics II**  
**MTH182 – Section 03 – Spring 2015**

**Problem set 4**  
**Functions**

Reading: Discrete Mathematics, first edition, section Sections 5.3  
 Section 5.3, 1, 3, 7, 9, 11, 13, 25, 29, 31

**Section 5.3**

1. Let  $A$  be the set of all nonempty subsets of  $B = \{1, 2, 3\}$ . For each  $f$  described below, determine whether  $f$  is a function from  $A$  to  $B$ . For  $S \in A$ ,
  - (a)  $f(S) = |S|$
  - (b)  $f(S) = |S| - 1$
  - (c)  $f(S) = 1/|S|$
  - (d)  $f(S)$  is the sum of the elements in  $S$ .
  - (e)  $f(S)$  is the largest element in  $S$ .
  - (f)  $f(S)$  is the absolute value of the difference of the largest element and the smallest element in  $S$ .
  
3. Let  $A = \{a, b, c, d, e\}$  and  $B = \{x, y, z\}$  and let  $f = \{(a, x), (b, x), (c, z), (d, x), (e, z)\}$  be a function from  $A$  to  $B$ .
  - (a) Determine the domain, codomain, and range of  $f$ .
  - (b) Determine the image of  $d$ .
  - (c) Determine whether  $y$  is an image.
  - (d) Determine  $f(X)$  where  $X = \{a, c, d\}$ .
  - (e) Give an example of a function  $g$  from  $B$  to  $A$ .
  
7. Prove that there exist nonempty sets  $A$  and  $B$ , a function  $f : A \rightarrow B$  and subsets  $A_1$  and  $A_2$  of  $A$  such that  $f(A_1 \cap A_2) \neq f(A_1) \cap f(A_2)$ .
  
9. Let  $A = \{a, b\}$  and  $B = \{0, 1, 2\}$ . Determine all functions from  $A$  to  $B$ .
  
11. The graph of  $x = y^2$  is a parabola, which is drawn in Figure 5.15 in the book. For each  $x \in \mathbb{R}$ , let  $y = f(x)$  be a real number such that  $(x, y)$  lies on the graph shown in Figure 5.15. Is  $f$  a function from  $\mathbb{R}$  to  $\mathbb{R}$ ?
  
13. A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = x(4 - x)$ . Find the domain, codomain, and range of  $f$ .
  
25.
  - (a) Is  $f$  a function from  $\mathbb{R}$  to  $\mathbb{R}$  if  $f(x) = \frac{1}{x^2-1}$  for each  $x \in \mathbb{R}$ ?
  - (b) Is  $f$  a function from  $\mathbb{Z}$  to  $\mathbb{Z}$  if  $f(n) = \frac{1}{n^2+1}$  for each  $n \in \mathbb{Z}$ ?
  - (c) Is  $f$  a function from  $\mathbb{Z}$  to  $\mathbb{N}$  if  $f(n) = |-\sqrt{n^2}|$  for each  $n \in \mathbb{Z}$ ?

- 29.** Let  $A = \{a, b, c, d\}$  and  $B = \{w, x, y, z\}$ . Consider the functions  $f : A \rightarrow A$  and  $g : A \rightarrow B$ , where  $f = \{(a, c), (b, a), (c, a), (d, b)\}$  and  $g = \{(a, 1), (b, 3), (c, 2), (d, 1)\}$ . Determine the following: (a)  $(g \circ f)(d)$ . (b)  $g \circ f$ . (c)  $f \circ f$ .
- 31.** Two functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  are defined by  $f(x) = 3x^2 + 1$  and  $g(x) = 5x - 3$  for all  $x \in \mathbb{R}$ . Determine the following. (a)  $(g \circ f)(0)$  and  $(f \circ g)(0)$ . (b)  $g \circ f$  and  $f \circ g$ .