Mathematics Department, University of Massachusetts Dartmouth

Discrete Mathemtics II

MTH182 – Section 03 – Spring 2015

Problem set 3

Equivalence relations

Reading: Discrete Mathematics, first edition, section Sections 5.2
Section 5.2, 1, 3, 5, 7, 9, 11, 15

Section 5.2

1. A relation R is defined on $\mathbb{N} \times \mathbb{N}$ by $(a, b)\ R\ (c, d)$ if $a + d = b + c$.
(**a**) Show that R is an equivalence relation.
(**b**) Describe the equivalence classes $[(3, 1)]$, $[(5, 5)]$, $[(4, 7)]$.

3. Let R be an equivalence relation on the set $S = \{a, b, c, d, e, f\}$. If the distinct equivalence classes are $\{a, d\}$, $\{b, f\}$, and $\{c, e\}$, what is R?

5. An equivalence relation R on the set $S = \{1, 2, 3, 4, 5, 6\}$ results in three distinct equivalence classes. Given that (a) $3 \in [4] \cap [5]$, (b) $[2] \cap [6] = \varnothing$, and (c) $1 \in [3]$, what is R?

7. Let R be a relation defined on $\mathbb{Z}$ by $a\ R\ b$ if $a + b = 0$ or $a - b = 0$.
(**a**) Determine whether R is an equivalence relation.
(**b**) If R is an equivalence relation, then describe the distinct equivalence classes.

9. A relation R is defined on the set $\mathbb{Z}$ of integers by $a\ R\ b$ if $11a - 5b$ is even.
(**a**) Show that R is an equivalence relation.
(**b**) Describe the distinct equivalence classes resulting from R.

11. A relation R is defined on $\mathbb{Z} \times \mathbb{Z}$ by $(a, b)\ R\ (c, d)$ if $abcd$ is even. Is R an equivalence relation?

15. Let S be a nonemtpy set and let $\mathcal{P} = \{S_1, S_2, \ldots, S_k\}$ be a partition of S, where $k \geq 1$. Define a relation R on S by $a\ R\ b$ if $a, b \in S_i$ for some i with $1 \leq i \leq k$.
(**a**) Prove that R is an equivalence relation.
(**b**) Describe the distinct equivalence classes resulting from R.