Mathematics Department, University of Massachusetts Dartmouth

Discrete Mathemtics II

MTH182 – Section 03 – Spring 2015

Problem set 2

Strong induction and relations

Reading:	Discrete Mathematics, first edition, section Sections 4.3, 4.4, 5.1 Section 4.3: 1 Section 4.4: 1, 3 Section 5.1: 1, 3, 5, 7, 9

Section 4.3

1. Write out the first four terms of the sequence $\{a_n\}$, for which $a_n = (-1)^{n+1}\frac{2n+3}{3\cdot 2^{n-1}}$.

Section 4.4

1. A sequence $\{a_n\}$ is defined recursively by $a_1 = 1$ and $a_n = 2a_{n-1}$ for $n \geq 2$.

(**a**) Determine a_2, a_3, a_4, and a_5.

(**b**) Conjecture a formula for a_n for each positive integer n.

(**c**) Verify the conjecture in part (b).

3. A sequence a_1, a_2, a_3, ... of integers is defined recursively by $a_1 = 4$, $a_2 = 7$, and $a_n = 2a_{n-1} - a_{n-2} + 2$ for $n \geq 3$. Prove that $a_n = n^2 + 3$ for every positive integer n.

Section 5.1

1. Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. Give an example of a relation R from A to B such that 1 is related to 5, but 2 is not related to 4.

3. For $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$, the set $R = \{(a, y), (a, z), (b, y), (c, x), (c, z)\}$ of ordered pairs is a relation from A to B. Describe this relation by a diagram.

5. Let $A = \{1, 2, 4, 5\}$ and $B = \{2, 3, 5\}$. A relation R from A to B is defined by $a\ R\ b$ if $a + b$ is prime. Express R as a set of ordered pairs.

7. The following are relations on the set $S = \{1, 2, 3, 4\}$. Which of the properties reflexive, symmetric, and transitive does each relation possess?

(**a**) $R_1 = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$

(**b**) $R_2 = \{(1, 2), (1, 3), (1, 4)\}$

(**c**) $R_3 = \{(1, 1), (1, 2), (2, 2), (2, 3), (3, 3), (3, 4), (4, 4), (4, 1)\}$

(**d**) $R_4 = \{(1, 2), (3, 4)\}$

(**e**) $R_5 = \{(1, 1), (2, 2), (3, 3), (3, 1), (4, 4)\}$

9. The relation R is defined on the set $\mathbb{R}$ of real numbers by $x\ R\ y$ if $x - y \geq 0$, that is, $x\ R\ y$ if the point (x, y) in the plane lies on or to the right of the line $y = x$.

(**a**) Given an example of two real numbers a and b such that $a\ R\ b$ and two real numbers c and d such that $c \not R\ d$.

(**b**) Which of the properties reflexive, symmetric, and transitive does R possess?