Mathematics Department, University of Massachusetts Dartmouth

Discrete Mathemtics II
MTH182 – Section 03 – Spring 2015
Problem set 11
Permutations and Combinations

Reading: Discrete Mathematics, first edition, section Sections 8.4, 9.1
Section 8.4: 1, 3, 5, 7, 9, 11, 13, 15, 17, 23, 29
Section 9.1: 1, 3, 5, 7, 9, 11, 13, 15, 17, 27

Section 8.4

1. Compute (a) $\frac{10!}{7!\,3!}$, (b) $P(8,2)$, (c) $\frac{6!}{0!}$, (d) $\frac{P(7,3)}{P(7,4)}$, (e) $\frac{n!}{(n-1)!}$.

3. Compute (a) $P(7,2)$ and $C(7,2)$, (b) $P(8,3)$ and $C(8,3)$, (c) $P(9,4)$ and $C(9,4)$, (d) $P(10,10)$ and $C(10,10)$.

5. How many different arrangements are there of the letters in the word "string"?

7. How many different ways are there of selecting 5 people form a group of 7 and seating them in a row of 5 chairs?

9. In how many orders can 4 married copules be seated in a row of 8 chairs if every one must sit next to his or her spouse?

11. How many subset of $\{a, b, c, d, e, f\}$ contain exactly three elements?

13. How many different 3-member committees can be formed form a group of 10 people?

15. How many subsets of $\{1, 2, 3, 4, 5, 6\}$ contain two or more elements?

17. How many 3-element subsets of $\{1, 2, \ldots, 10\}$ contain only even integers?

23. A total of 20 people apply for a university position.

(**a**) In how many ways can 8 of the 20 applicants be selected to form a "short list" of applicants?

(**b**) In how many ways can 5 candidates out of 8 be selected on a "short list" to be interviewed?

(**c**) In how many ways can 3 of the 5 candidates who are interviewed be ordered for ranking purposes?

29. Prove that $rC(n,r) = nC(n-1,r-1)$ for $1 \leq r \leq n$.

Section 9.1

1. Another property of the Pascal triangle concerns 4 numbers lying within certain rhombuses (parallelograms with equal sides). See Figure 9.7 in the book.

(**a**) Compute the sum of these numbers. Based on this, make a guess and show that your guess is correct.

(**b**) For each such rhombus, compute the product of the elements in the upper left and lower right positions minus the product of the remaining two elements. Based on this, make a guess and show that your guess is correct.

3. Is the number 22 one of the numbers in the Pascal triangle?

5. Expand $(x-y)^6$, giving precise coefficients.

7. Expand $(x+y)^8$, giving precise coefficients.

9. What is the exact term in $(x+y)^{10}$ containing x^4?

11. What is the exact term in $(2x^2+\frac{y}{2})^8$ containing x^6?

13. What is the exact coefficient of x^4y^3 in $(2x-3y)^7$?

15. What is the exact term in $(2x^2-y)^8$ containing x^6?

17. One term in the expansion of $(2x-\frac{1}{2x^2})^{10}$ is cx for some constant c. Determine c.

27. Show that $\sum_{r=0}^{n} \left(\begin{array}{c} n \\ r \end{array} \right) 2^{2n-2r} = 5^n$.