Section 8.4

1. Compute (a) $\frac{10!}{7!3!}$, (b) $P(8,2)$, (c) $\frac{6!}{2!}$, (d) $\frac{P(7,3)}{P(7,4)}$, (e) $\frac{n!}{(n-1)!}$.

3. Compute (a) $P(7,2)$ and $C(7,2)$, (b) $P(8,3)$ and $C(8,3)$, (c) $P(9,4)$ and $C(9,4)$, (d) $P(10,10)$ and $C(10,10)$.

5. How many different arrangements are there of the letters in the word “string”?

7. How many different ways are there of selecting 5 people from a group of 7 and seating them in a row of 5 chairs?

9. In how many orders can 4 married couples be seated in a row of 8 chairs if every one must sit next to his or her spouse?

11. How many subset of \{a, b, c, d, e, f\} contain exactly three elements?

13. How many different 3-member committees can be formed from a group of 10 people?

15. How many subsets of \{1, 2, 3, 4, 5, 6\} contain two or more elements?

17. How many 3-element subsets of \{1, 2, \ldots, 10\} contain only even integers?

23. A total of 20 people apply for a university position.
   (a) In how many ways can 8 of the 20 applicants be selected to form a “short list” of applicants?
   (b) In how many ways can 5 candidates out of 8 be selected on a “short list” to be interviewed?
   (c) In how many ways can 3 of the 5 candidates who are interviewed be ordered for ranking purposes?

29. Prove that $rC(n,r) = nC(n-1,r-1)$ for $1 \leq r \leq n$. 

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Section 9.1

1. Another property of the Pascal triangle concerns 4 numbers lying within certain rhombuses (parallelograms with equal sides). See Figure 9.7 in the book.
   (a) Compute the sum of these numbers. Based on this, make a guess and show that your guess is correct.
   (b) For each such rhombus, compute the product of the elements in the upper left and lower right positions minus the product of the remaining two elements. Based on this, make a guess and show that your guess is correct.

3. Is the number 22 one of the numbers in the Pascal triangle?

5. Expand \((x - y)^6\), giving precise coefficients.

7. Expand \((x + y)^8\), giving precise coefficients.

9. What is the exact term in \((x + y)^{10}\) containing \(x^4\)?

11. What is the exact term in \((2x^2 + \frac{y}{2})^8\) containing \(x^6\)?

13. What is the exact coefficient of \(x^4y^3\) in \((2x - 3y)^7\)?

15. What is the exact term in \((2x^2 - y)^8\) containing \(x^6\)?

17. One term in the expansion of \((2x - \frac{1}{x^2})^{10}\) is \(cx\) for some constant \(c\). Determine \(c\).

27. Show that \(\sum_{r=0}^{n} \binom{n}{r} 2^{2n-2r} = 5^n\).