Mathematics Department, University of Massachusetts Dartmouth

Discrete Mathemtics II

MTH182 – Section 03 – Spring 2015

Problem set 10

The addition and multiplication principles

Reading:	Discrete Mathematics, first edition, section Sections 8.1, 8.2, 8.3 Section 8.1: 1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25 Section 8.2: 1, 3, 7, 9, 11 Section 8.3: 1, 3, 5, 7, 9, 11, 13, 17, 19, 21

Section 8.1

1. On a particular afternoon a student plans to take her nbiece to a movie followed by dessert. The possibilities for the movie are an animated movie, a comedy, and an adventure movie, while the choices for dessert are hot fudge sundae, strawberry shortcake, apple pie, and chocolate milkshake. How many possibilities are there for the afternoon events?

3. Each seat in an arena is labeled with a letter of the alphabet followed by a positive integer not exceeding 50. What is the largest number of seats that can be labeled differently?

5. At a certain restaurant, a dinner starts with soup, salad, or juice, followed by the main course (steak, chicken, shrimp, or fish), followed by dessert (pie or cake). How many possibilities for dinner are there?

7. Let A and B be two sets with $|A| = 5$ and $|B| = 6$.

(**a**) How many different functions from A to B are there?

(**b**) How many different functions from B to A are there?

(**c**) How many different one-to-one functions from A to B are there?

(**d**) How many different one-to-one functions from B to A are there?

9. In how many orders can 4 married couples be seated in a row of 8 chairs if everyone must sit next to his or her spouse and no two people of the same sex can sit next to each other?

11. How many different 4-digit numbers can be formed from the digits 1, 2, 3, 4, 5, 6, if

(**a**) digits can be repeated?

(**b**) digits cannot be repeated?

15. It is decided to have dinner at a Chinese, Italian, or Mexican restaurant. If there are 7 possible Chinese restaurants, 5 possible Italian restaurants, and 4 possible Mexican restaurants, what is the total number of possible choices for restaurants?

17. At a certain university, a telephone number begins with 355 or 357 and is then followed by four digits.

(**a**) How many telephone numbers are possible?

(**b**) How many telephone numbers have no repeated digits?

19. How many different 10-bit strings begin and end with the same 5-bit string?

21. How many different 7-bit strings begin with 11 or 0011?
23. How many different 10-bit strings begin with 1011 or 001100?
25. A license plate consists of a sequence of three letters followed by three digits, or three digits followed by three letters. How many different license plates are there?

Section 8.2

1.
(**a**) How mnay different 8-bit strings begin with 010 or begin with 11?
(**b**) How many different 8-bit strings begin with 010 or end with 11?

3.
(**a**) How many different 10-bit strings begin with 1010 or begin with 0111?
(**b**) How many different 10-bit strings begin with 1010 or end with 0111?
(**c**) How many different 10-bit strings begin with 1010 and end with 0111?

7. How many different 8-bit strings have 00 as consecutive bits as either (a) its first and second bits, (b) its third and fourth bits, (c) its fifth and sixth bits, or (d) its seventh and eighth bits?
9. There are 64 students who take at least one class during the morning, afternoon, or evening. How many of these students take at least one morning class, at least one afternoon class, *and* at least one evening class if

- 29 students take morning classes,
- 38 students take afternoon classes,
- 35 students take evening classes,
- 19 students take monring and afternoon classes,
- 8 students take morning and evening classes, and
- 19 students take afternoon and evening classes?

9. A total of 60 students studying in a mathematics library are interviewed as to what courses they are taking this semester. Here are the responses:

- 26 students are taking mathematics.
- 35 students are taking computer science.
- 28 students are taking statistics.
- 15 students are taking mathematics and computer science.
- 13 students are taking mathematics and statistics.
- 18 students are taking computer science and statistics
- 7 students are taking all three.

(**a**) How many students are taking none of these courses?
(**b**) How many studenst are taking exactly one of the courses?
(**c**) How many students are taking exactly two of the courses?

Section 8.3

1. Let n be a positive integer. Show that $n = \lceil n/2 \rceil + \lfloor n/2 \rfloor$.

3. According to the Pigeonhole Principle, if 420 notebooks are distributed to 40 students, there must be at least one student who receives at least n notebooks for some positive integer n, but there is not guarantee that any student will receive $n + 1$ notebooks. What is n?

5. In a set of 27 English words, what is the largest number of words that must begin with the same letter?

7. A bowl contains 9 marbles, each of which is red, blue, or green. Show that at least 3 of these marbles are red, 3 are blue, or 5 are green.

9. There are 115 books to be placed on 9 bookshelves. What is the smallest possible number of books that can be placed on the shelf with the largest number of books?

11. A conference room contains 8 tables and 107 chairs. What is the smallest possible number of chairs at the table having the most chairs?

13. A bowl contains a large number of red, blue, and yellow marbles. What is the fewest number of marbles that need to be randomly selected form the bowl to be guaranteed that 9 marbles of the same color are chosen?

17. Two teams play baseball games until one team wins four games. What is the fewest number of games they must play to be guaranteed that one team will win four games?

19. Each piece of fruit in a fruit basket is either an apple, a banana, an orange, a pear, or a peach. How many pieces of fruit must be in the basket to guarantee that there is at least one apple, at least two bananas, at least three oranges, at least four pears, or at least five peaches?

21. A young lady has eleven pairs of shoes that she wears on special occasions. Each pair is either black, brown, or white. Show that she either has at least four pairs of black shoes, at least four pairs of brown shoes, or at least five pairs of white shoes.