Project 4 + Final Project
Numerical Experiments
2D Navier-Stokes Equation with Pseudospectral Methods
Submission procedure: through mycourses.

Name : _____

One of many active research areas in fluid dynamics is to numerically solve the 2D Navier-Stokes (N-S) equation written in the stream function $\psi(x,y,t)$ formulation. In that case, the N-S equation is written as

$$(\Delta\psi)_t + \psi_y(\Delta\psi)_x - \psi_x(\Delta\psi)_y = \nu\Delta\Delta\psi \quad \text{in } \Omega$$

with boundary conditions

$$\psi = 0 \text{ and } \frac{\partial\psi}{\partial\mathbf{n}} = 0 \quad \text{at } \partial\Omega.$$

Some of you may still remember when Prof. Dalia Fishelov from Tel-Aviv University gave a talk about this problem at UMass Dartmouth Computational Science Seminar back in January 2013.

To Do Lists:

1. Try your best to numerically simulate the above problem for the case of $\nu = 10^{-2}$ with pseudospectral methods in the domain $[-1,1]\times[-1,1]$. As an initial condition, you may take $\psi(x,y) = -\frac{1}{2}\sin(\pi x)\sin(\pi y)$ or a function of your own choice.
2. Use contour plot command in **MATLAB** to plot $\psi(x,y,t)$ at times $t = 0.25,0.5,0.75,1$.
3. Use ψ to compute velocity functions u, v, where $u = \psi_y$ and $v = -\psi_x$. Use contour plot command in **MATLAB** to plot velocity functions $u(x,y,t)$ and $v(x,y,t)$ at times t=0.25,0.5,0.75,1.

Hints and Rules:

1. You are free to use **cheb.m**, **DMSUITE**, or **chebfun** to create differentiation matrices. Use techniques learned in class regarding how to impose multiple boundary conditions.
2. Feel free to experiment with any explicit or implicit methods of your own choice to march in times. You can also use MATLAB ode solver.
3. Feel free to discuss your MATLAB codes with me during my office hours.
4. Your instructor (me) might be having trouble doing this problem too. Let us have some fun and struggle together.