Name_____ **Student ID #** _____

Instructor_____ **Lab Period** _____ **Date Due** _____

Lab 13
Polar Graphs

Objectives

1. To investigate the different representations of the same polar curve.
2. To develop an understanding of how polar graphs are drawn.

In this lab, all graphing will be done using polar coordinates, thus, we need to set up *TEMATH* by doing the following:

- Select **Polar** from the **Graph** menu.

Exploration 1 *Finding Different Representations of the Four-Leaf Rose*

In this exploration, we want to investigate the order in which the leaves of a four-leaf rose are drawn. To start, let's plot the four-leaf rose $r = \sin(2t)$ over the interval $0 \le t \le 2\pi$ by doing the following:

- Enter and plot the polar equation $r = \sin(2t)$ over the interval $0 \le t \le 2\pi$.
- Click the **Polar Tracker** tool  in the **Graph** window.
- Position the arrow cursor over the large dot  of the Polar Tracker tool. Press and hold down the mouse button. Drag the tracker in a counterclockwise circular direction and determine the order in which the four leaves of the rose are drawn by observing how the dot moves along the curve.

Note that as t increases from 0 to 2π, the four leaves of the rose are drawn in the order: Quadrant 1, Quadrant 4, Quadrant 3, and Quadrant 2. Since it is possible to have different algebraic equations representing the same polar curve, it is natural to ask the question: "Is it possible to find a polar equation whose graph is this same four-leaf rose but the leaves are drawn in a different order?" To try to answer this question, let's do the following:

- Enter and plot the polar equation $r = |\sin(2t)|$ over the interval $0 \le t \le 2\pi$. The *TEMATH* expression for $|\sin(2t)|$ is **abs(sin(2t))**.
- Use the **Polar Tracker** to determine the order in which the four leaves are drawn.

1. List the order of the quadrants in which the four leaves of the rose are drawn as t increases from 0 to 2π. ……

   ……

2. Find the polar equation for the four-leaf rose whose leaves are drawn in the order: Quadrant 3, Quadrant 2, Quadrant 1, and Quadrant 4. ……

   ……

3. Find the polar equation for the four-leaf rose whose leaves are drawn in the order: Quadrant 3, Quadrant 4, Quadrant 1, and Quadrant 2. ……

   ……

4. a) Why did the order change in questions 2 and 3? ……

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   ……

   b) Are there any other possible orderings? Explain why or why not. ……

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5. Is there any condition that we could change in this exploration to get some different orderings? Explain. ……

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Exploration 2 *The Cardiod*

The graph of the polar equation $r = a + b\cos(t)$, where $|a| = |b|$, is called a cardiod. In this exploration, we want to study how the parameters *a* and *b* affect the orientation of the cardiod. Let's begin by doing the following:

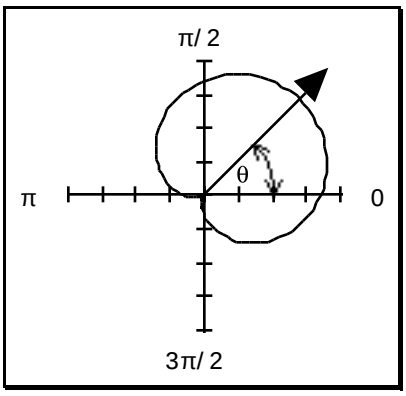
- Enter and plot the polar equation $r = 2 + 2\cos(t)$ over the interval $0 \le t \le 2\pi$.
- Use the **Polar Tracker** to determine how the cardiod is drawn.

1. Explain why this graph is called a "cardiod". ……

……

……

You can measure the pointing direction of the cardiod by using the value of the angle θ (measured in radians) as is shown in the following figure:



2. a) What is the value of the angle θ that measures the pointing direction of the cardiod $r = 2 + 2\cos(t)$? ……

b) What is the value of *t* for which $r = 4$?……

What happens to the graph of the cardiod if we change the value of *a* from 2 to –2? To find the answer to this question, do the following:

- Enter and plot the polar equation $r = -2 + 2\cos(t)$ over the interval $0 \le t \le 2\pi$.
- Use the **Polar Tracker** to determine how the cardiod is drawn.

3. Is there any difference between the graph of $r = -2 + 2\cos(t)$ and the graph of

$r = 2 + 2\cos(t)$?…… If yes, explain?……

……

The next part of this exploration is to determine the relationship between the sign of the number *b* and the orientation of the cardiod. To begin, let's change the value of *b* from 2 to –2 and plot a new cardiod by following these instructions:

- Enter and plot the polar equation $r = 2 - 2\cos(t)$ over the interval $0 \le t \le 2\pi$.

4. a) What is the value of the angle θ that measures the pointing direction of the cardiod

$r = 2 - 2\cos(t)$? ……

b) What is the value of *t* for which $r = 4$?……

c) What affect does changing the sign of *b* have on the pointing direction (orientation)

of the cardiod? ……

……

……

By this point you should have a good understanding of how the sign of *a* and *b* affect the orientation or drawing of the cardiod. Now here is a really tough question: Can you find a polar equation for a cardiod pointing in the direction θ for any value of θ?

5. a) Find a polar equation for the cardiod pointing in the direction $\theta = \pi/4$ ……

……

b) Find a polar equation for the cardiod pointing in the direction $\theta = \pi/2$ ……

……

c) Find a polar equation for the cardiod pointing in the direction $\theta = -3\pi/4$ ……

……

d) Find a polar equation for the cardiod pointing in the direction $\theta = 2\pi/3$ ……

……