Name________________________________ **Student ID #** _________________________

Instructor___________________________ **Lab Period** _____ **Date Due** ___________

Lab 9
Exponential Functions

Objectives

1. To develop an ability to interpret the important characteristics of the graphs of exponential functions.
2. To become familiar with applications of the exponential function.

Locating Points on a Curve

You can use the Rectangular Tracker tool  to locate points on a curve. You can also use the mouse to find the approximate values of the coordinates of any point in the Graph window by doing the following:

- Click in the Graph window to make it the active window.
- As you move the mouse over the Graph window, the coordinates of the mouse location are shown in the bottom two cells of the Domain & Range window.

Exploration 1 *Learning Curve*

A learning curve describes the speed with which a person becomes competent at performing a particular task. In this case, the learning curve can be represented by the exponential function $y(t) = a_1 - a_2 b^{ct}$, where a_1, a_2, b and c are constants and t represents time. As an example, let's consider a new employee Smith who was hired to inspect electronic circuit boards that are to be installed in personal computers. After a brief training period, Smith begins the job. After each additional day on the job, Smith performs more efficiently and the number of circuit boards Smith inspects each day after the first day on the job is given by the exponential learning curve

$$y(t) = 50 - 40(0.6)^{t/5}$$

where

t = the number of days on the job,

and

$y(t)$ = the number of circuit boards inspected on the t-th day on the job.

Let's use *TEMATH* to analyze this example by doing the following:

- Select **Variables** from the **Options** menu and change the name of the x-variable to $\boldsymbol{t}$.

- Click *off* **autoscaling** in the **Domain & Range** window.
- Set the plotting domain to $\mathbf{0 \le t \le 50}$ and set the plotting range to $\mathbf{0 \le y \le 60}$.
- Enter and plot $y(t)$. The *TEMATH* expression for $y(t)$ is **50 – 40(0.6)^(t/5)**.

In the following questions, assume that the employee works a five day work week and use *TEMATH*'s Rectangular Tracker tool to obtain the needed estimates.

1. Use the learning curve to estimate how many circuit boards Smith inspected on:

   a) the first day on the job. ……………………………………………………………

   b) the fifth day on the job (the end of the first week on the job). ………………………

   c) the twentieth day on the job (the end of the first month on the job). …………………

   d) the fiftieth day on the job. …………………………………………………………

   e) Use the learning curve to describe in words how the competence of Smith increases as more days are spent on the job.……………………………………………………

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   f) What appears to be the maximum number of circuit boards that Smith will be able to inspect on any particular day? ………………………………………………………

Exploration 2 *Spread of a Rumor*

Suppose the speed at which a rumor spreads is given by the function

$$p(t) = \frac{1}{1 + 999e^{-t}},$$

where $p(t)$ is the proportion of the population that has heard the rumor after t days. Let's investigate how a rumor spreads by doing the following:

- Click *on* **autoscaling** in the Domain & Range window and set the plotting domain to $0 \le t \le 15$.
- Enter and plot the rumor function $p(t)$. The *TEMATH* expression for $p(t)$ is **1/(1+999 exp(–t))**.

1. Use the rumor curve to answer the following:

   a) How many days does it take for 50% of the population to hear the rumor? …………

   b) How many days does it take for 90% of the population to hear the rumor? …………

2. Give a description in words of how fast the rumor spreads over time. ……………………

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Exploration 3 *Memorizing Facts*

A psychologist claims that the number of facts $y(t)$ that a student can memorize in t hours of studying is given by

$$y(t) = 40 - 30(0.2)^{t/2}$$

where t is the number of hours the student studied.

To use *TEMATH* to help you analyze this problem, do the following:

- Click *off* **autoscaling** in the Domain & Range window.
- Set the plotting domain to $\mathbf{0 \le t \le 5}$ and set the plotting range to $\mathbf{0 \le y \le 40}$.
- Enter and plot $y(t)$.

A student has an exam scheduled for tomorrow. The student needs to memorize forty facts before the exam.

1. a) If you were this student, how long would you study for the test? ……………………

   b) Why? ……………………………………………………………………………………

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Exploration 4 *The Exponential Growth Model*

In theory, the exponential growth model can be used to model population growth under certain assumptions. For example, if the size of a population at time $t = 0$ is P_0 and the continuous rate of growth of the population is r, then the population at time t is given by the exponential model

$$P(t) = P_0 e^{rt}$$

Let's test this model on some real data. We will use the US Population Census Data for the years 1790 to 1990.

US Population for the Years 1790-1990			
Year	**Population (Millions)**	**Year**	**Population (Millions)**
1790	3.9	1900	76.0
1800	5.3	1910	92.0
1810	7.2	1920	105.7
1820	9.6	1930	122.8
1830	12.9	1940	131.7
1840	17.1	1950	150.7
1850	23.1	1960	179.0
1860	31.4	1970	205.0
1870	38.6	1980	226.5
1880	50.2	1990	248.7
1890	62.9		

To begin this exploration, let's enter the population data for the years 1790-1860 into *TEMATH* by following these instructions:

- Select **New Data Table — Keyboard Entry...** from the **Work** menu.
- Before we enter the data, we need to code the data so that the year 1790 is represented by $t = 0$, the year 1800 is represented by $t = 10$, and so on as is shown below:

t	***y***
0	3.9
10	5.3
20	7.2
30	9.6
40	12.9
50	17.1
60	23.1
70	31.4

Enter this data into the **Data Table** window.

- Click the **close box** of the **Data Table** window and **Add** the table to the **Work** window.
- Click *off* autoscaling, set the plotting domain to $\mathbf{0 \le t \le 70}$ and set the plotting range to $\mathbf{0 \le y \le 40}$.
- Select **Plot** from the **Graph** window.

There is a mathematical procedure called “The Method of Least Squares” that is used to *fit* mathematical functions to experimental data. In this exploration, we want to fit an exponential function of the form $y(t) = y_o e^{rt}$ to the data. To do this in *TEMATH*, select **Least Squares — Natural Exponential** from the **Tools** menu. The fitted natural exponential function will be written into the first cell of the Work window and its graph will be overlaid on top of the plot of the data.

1. a) What is the equation of the Least Squares fitted exponential model?..…………………

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b) Is the exponential model a good fit to the data? Explain..……………………………

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c) What is the estimated continuous growth rate *r* for the years 1790-1860?.……………

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Next, let's enter the population data for all the years from 1790-1990.

- Select **New Data Table — Keyboard Entry...** from the **Work** menu.
- Code the data as before with the year 1790 being represented by $t = 0$, 1800 being represented by $t = 10$, and so on.
- Click the **close box** of the **Data Table** window and **Add** the table to the **Work** window.
- Click *off* autoscaling, set the plotting domain to $\mathbf{0 \le t \le 200}$ and set the plotting range to $\mathbf{0 \le y \le 250}$.
- Select **Plot** from the **Graph** window.
- Click in the **Work** window and select the Least Squares fitted natural exponential model obtained in question 1 above. Select **Overlay Plot** from the **Graph** menu.

2. How well does the exponential model for the years 1790-1860 fit the population data for the years 1790-1990? Explain.……………………………………………………………
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3. Can you think of some historical event that took place in the 1860-1870 decade that might have affected the population growth rate? …………………………………………
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Finally let's find the Least Squares exponential model for the years from 1790-1990 by doing the following:

- Click in the **Work** window and select the Data Table containing all the population data.
- Select **Plot** from the **Graph** menu.
- Select **Least Squares — Natural Exponential** from the **Tools** menu.

4. a) What is the Least Squares natural exponential model for the years from 1790-1990?.
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b) Is this exponential model a good fit to the data? ……………………………………
c) If the exponential model is not a good fit, give some reasons why it is not a good fit and, also, why the exponential growth model theory fails..……………………………
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