**Name** _____ **Student ID #** _____

**Instructor** _____ **Lab Period** _____ **Date Due** _____

# Lab 5
# Continuity

## Objectives

**1**. To visually represent the concept of continuity.
**2**. To develop an informal intuition for continuity.

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**Continuity**

A fuction is *continuous* on an interval if its graph has no holes, jumps, or breaks in that interval. More specifically, a function is *continuous at* $x = a$ if

1) $f(a)$ exists ($a$ is in the domain of $f$)
2) $\lim_{x \to a} f(x)$ exists
3) $\lim_{x \to a} f(x) = f(a)$

Additionally, a function $f$ is *continuous from the left at the number* $a$ if

1) $f(a)$ exists ($a$ is in the domain of $f$)
2) $\lim_{x \to a^-} f(x)$ exists
3) $\lim_{x \to a^-} f(x) = f(a)$

and a function $f$ is *continuous from the right at the number* $a$ if

1) $f(a)$ exists ($a$ is in the domain of $f$)
2) $\lim_{x \to a^+} f(x)$ exists
3) $\lim_{x \to a^+} f(x) = f(a)$

A function $f$ is said to be *continuous on the open interval* $(a, b)$ if $f$ is continuous at every number in the interval. The open interval $(a, b)$ may be $(-\infty, b)$, $(a, \infty)$, or $(-\infty, \infty)$. A function $f$ is said to be *continuous on the closed interval* $[a, b]$ if $f$ is continuous on $(a, b)$ and if it is also continuous from the right at $a$ and continuous from the left at $b$.

If a function $f$ is not continuous at $x = a$, then $f$ is said to be *discontinuous* at $x = a$.

## Exploration 1 Finding Discontinuities

The goal of this exploration is for you to develop a good intuition for continuous functions and to be able to recognized and classify the discontinuities of a function. You will be given a set of functions to examine and you will be asked to find the intervals on which they are continuous, the numbers at which they are discontinuous, and to classify the types of discontinuities. In the following exercises, you can use *TEMATH*'s Rectangular Tracker tool to help visualize where a function is continuous and where it is discontinuous.

Begin this exploration by doing the following:

- Plot $f(x) = \begin{cases} \dfrac{1}{(x+3)(x+4)} & \text{if } x < -1 \\ x & \text{if } x \geq -1 \end{cases}$ on the interval $-5 \leq x \leq 5$. In *TEMATH*, this piecewise function is written as **1/((x + 4)(x + 3)) if x < –1; x**.
- Click the **Rectangular Tracker Tool**  to make it active.
- Place the cursor on the upward pointing arrow ▲ at the *left end* of the *x*-axis.
- Press and hold down the mouse button and drag the arrow toward the right along the *x*-axis.

Remember that the graph of a continuous function is smooth and it has no holes, jumps, or breaks. As you track the graph with the Rectangular Tracker, observe whether the graph is smooth or whether there are points of discontinuiy where the graph has a hole, a jump, or a break.

As you drag the Rectangular Tracker along the *x*-axis, the resolution of the computer monitor may make it impossible to move the tracker to the exact value of *x* that you want, for example, it may be impossible to move the tracker to the point on the *x*-axis where *x* is exacly equal to 1. However, you can use the Domain & Range window to move the tracker to a more accurate position by doing the following:

- Click in the **Domain & Range** window to make it active.
- Enter the value of *x* that you want to move the tracker to into the **x-cell** in the bottom portion of the **Domain & Range** window.
- Press the **Enter** key or click the **Enter** button in the **Domain & Range** window.

The tracker will move along the *x*-axis to the value of *x* that you entered. This method of moving the tracker is useful for investigating one-sided continuity.

1. a) List the values of $x$ at which $f$ is discontinuous, give a reason why $f$ is discontinuous at each of the values, and classify the type of discontinuity as a hole (removable discontinuity), a jump (jump discontinuity), or a break (the number is not in the domain of $f$).……

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b) What are the values of $x$ (if any) at which $f$ is continuous only from the left?……

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c) What are the values of $x$ (if any) at which $f$ is continuous only from the right?……

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d) List the intervals (open, closed, or half-open and half-closed) on which $f$ is continuous (be sure to consider the entire domain of the function and not just the interval over which the function is plotted)..……

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In the following example, drag the Rectangular Tracker carefully and very slowly by the value $x = 1$ on the $x$-axis.

- Plot $f(x) = \begin{cases} \dfrac{\sin(x-1)}{x-1} & \text{if } x \neq 1 \\ 0.5 & \text{if } x = 1 \end{cases}$ on the interval $-\mathbf{3} \leq \boldsymbol{x} \leq \mathbf{5}$. In *TEMATH*, the piecewise function is written as **sin(x – 1)/(x – 1) if x ≠ 1; 0.5** (press the **Option =** key to get ≠).

2. a) List the values of $x$ at which $f$ is discontinuous, give a reason why $f$ is discontinuous at each of the values, and classify the type of discontinuity as a hole (removable discontinuity), a jump (jump discontinuity), or a break (the number is not in the domain of $f$).……

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b) What are the values of $x$ (if any) at which $f$ is continuous only from the left?……

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c) What are the values of $x$ (if any) at which $f$ is continuous only from the right?……

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d) List the intervals (open, closed, or half-open and half-closed) on which $f$ is continuous (be sure to consider the entire domain of the function and not just the interval over which the function is plotted)..……

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In the following example, consider the domain of $f(x)$ to be the same as the plotted domain $1 \le x \le 5$.

- Plot $f(x) = \dfrac{[[x]]}{x}$ on the interval $\mathbf{1 \le x \le 5}$. In *TEMATH*, the greatest integer function $[[x]]$ is written as **gint(x)**.

3. a) List the values of $x$ at which $f$ is discontinuous, give a reason why $f$ is discontinuous at each of the values, and classify the type of discontinuity as a hole (removable discontinuity), a jump (jump discontinuity), or a break (the number is not in the domain of $f$).……

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b) What are the values of $x$ (if any) at which $f$ is continuous only from the left?……

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c) What are the values of $x$ (if any) at which $f$ is continuous only from the right?……

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d) List the intervals (open, closed, or half-open and half-closed) on which $f$ is continuous (be sure to consider the entire domain of the function and not just the interval over which the function is plotted)..……

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- Plot $f(x) = \sqrt{1 - x^2}$ on the interval $-2 \le x \le 2$. In *TEMATH*, the square root function is **sqrt(x)** or √**(x)** (press **Option v** for √).

4. a) List the values of $x$ at which $f$ is discontinuous, give a reason why $f$ is discontinuous at each of the values, and classify the type of discontinuity as a hole (removable discontinuity), a jump (jump discontinuity), or a break (the number is not in the domain of $f$).……

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b) What are the values of $x$ (if any) at which $f$ is continuous only from the left?……

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c) What are the values of $x$ (if any) at which $f$ is continuous only from the right?……

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d) List the intervals (open, closed, or half-open and half-closed) on which $f$ is continuous (be sure to consider the entire domain of the function and not just the interval over which the function is plotted)..……

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- Plot $f(x) = \sqrt{x^3 - x}$ on the interval $-2 \le x \le 2$.

5. a) List the values of $x$ at which $f$ is discontinuous, give a reason why $f$ is discontinuous at each of the values, and classify the type of discontinuity as a hole (removable discontinuity), a jump (jump discontinuity), or a break (the number is not in the domain of $f$).……

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b) What are the values of $x$ (if any) at which $f$ is continuous only from the left?……

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c) What are the values of $x$ (if any) at which $f$ is continuous only from the right?……

……

d) List the intervals (open, closed, or half-open and half-closed) on which $f$ is continuous (be sure to consider the entire domain of the function and not just the interval over which the function is plotted)..……

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## Exploration 2 The Intermediate Value Theorem

**Intermediate Value Theorem**

Suppose that $f$ is continuous on the closed interval $[a,b]$ and let $N$ be any number strictly between $f(a)$ and $f(b)$. Then there exists a number $c$ in $(a,b)$ such that $f(c) = N$.

Why does the Intermediate Value Theorem require $f$ to be continuous on $[a,b]$? Why must $N$ be strictly between $f(a)$ and $f(b)$? To find the answers to these questions, let's do the following:

- Let $N = 3$ and plot $f(x) = 2 - x^2$ on the interval $\mathbf{0} \le \boldsymbol{x} \le \mathbf{5}$.

1. a) Is $f$ continuous on [0,5] ? ……. b) Is $N = 3$ strictly between $f(0)$ and $f(5)$?……

c) Does there exist a number $c$ in $(0,5)$ such that $f(c) = 3$? …… If so, then use the Rectangular Tracker Tool to estimate a value for $c$.……

- Let $N = 2$ and plot $f(x) = \begin{cases} 2x - 1 & \text{if } x < 1 \\ x^2 & \text{if } x \ge 1 \end{cases}$ on the interval $-\mathbf{2} \le \boldsymbol{x} \le \mathbf{3}$.

2. a) Is $f$ continuous on $[-2,3]$ ? …… b) Is $N = 2$ strictly between $f(-2)$ and $f(3)$? ……

c) Does there exist a number $c$ in $(-2,3)$ such that $f(c) = 2$ ? ……. If so, then use the Rectangular Tracker Tool to estimate a value for $c$. ……

- Let $N = -2$ and Plot $f(x) = x^2 - 9$ on the interval $\mathbf{-3 \le x \le 3}$.

3. a) Is $f$ continuous on $[-3,3]$? …… b) Is $N = -2$ strictly between $f(-3)$ and $f(3)$?…….

   c) Does there exist a number $c$ in $(-3,3)$ such that $f(c) = -2$?…… If so, then use the Rectangular Tracker Tool to estimate a value for $c$. ……

- Let $N = 2.5$ and plot $f(x) = [[x]] + (x - [[x]])^2$ on the interval $\mathbf{-3 \le x \le 3}$, where $[[x]]$ is the greatest integer function. In *TEMATH* the function expression is written as **gint(x) + (x – gint(x))^2**.

4. a) Is $f$ continuous on $[-3,3]$? …… b) Is $N = 2.5$ strictly between $f(-3)$ and $f(3)$?……

   c) Does there exist a number $c$ in $(-3,3)$ such that $f(c) = 2.5$?…… If so, then use the Rectangular Tracker Tool to estimate a value for $c$.……

## Exploration 3 Continuity of the Sum of Two Functions

| If $f$ and $g$ are continuous on the interval $(a,b)$, then $f + g$ is also continuous on $(a,b)$. |
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What happens if $f$ or $g$ or both are not continuous on $(a,b)$? Let's investigate.

- Click autoscaling *off*, set the domain to $\mathbf{-5 \le x \le 5}$, and set the range to $\mathbf{-10 \le y \le 10}$.
- Plot $f(x) = \begin{cases} x - 1 & \text{if } x < 1 \\ x & \text{if } x \ge 1 \end{cases}$.

1. Is $f$ continuous on the interval $(-5,5)$? ……

- Overlay the plot of $g(x) = \begin{cases} x + 2 & \text{if } x < 1 \\ x + 1 & \text{if } x \ge 1 \end{cases}$.

2. Is $g$ continuous on the interval $(-5,5)$? ……

- Select **New Function** from the **Graph** menu. If, for example, **y10(x)** and **y11(x)** are the *TEMATH* names for $f(x)$ and $g(x)$, enter **y10(x) + y11(x)** into the first cell of the **Work** window. This new function is $f + g$. **Overlay** the plot of this function.

3. Is $f+g$ continuous on the interval $(-5,5)$? ……

   • Plot $f(x)=\begin{cases}-1 & \text{if } x<0\\ x & \text{if } x\geq 0\end{cases}$.

4. Give an example of a function $g$ such that $f+g$ is continuous on the interval $(-5,5)$.

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5. Attach or sketch the graphs of $f$, $g$ and $f+g$ on the *same* set of axes.