Name_____ **Student ID #** _____

Instructor_____ **Lab Period** _____ **Date Due** _____

Lab 3
The Complete Graph

Objectives

1. To use the computer as a tool in sketching curves.
2. To distinguish the important characteristics of a curve.
3. To draw a complete graph.

Curve sketching (even with the help of a computer) can be difficult. For example, over what interval should you plot a function to observe all its interesting characteristics? No plotting program will simply give you the best interval. However, you can use a plotting program to gain insight into the behavior of the function; and, with some mathematical analysis, you can obtain a graph of the function showing its most interesting characteristics. *TEMATH* provides the following tools to help you find the important characteristics of the graph of a function.

Intercepts

To find an *x*-intercept (root or zero) of a function, click the **Single Root Finder** tool  and click in the **Graph** window near the *x*-intercept. The value of the *x*-intercept will be written into the **Report** window.

To find the *y*-intercept of a function, following these instructions: click the **Rectangular Tracker** tool  , click in the **Domain & Range** window to make it active, then enter **0** into the ***x*-cell** in the bottom portion of the window and press the **Enter** key or click the **Enter** button. The value of the *y*-intercept will be written into the *y*-cell in the bottom portion of the Domain & Range window and the Rectangular Tracker's dot will be positioned at the *y*-intercept in the Graph window.

Symmetry

To check if a function is an even function (symmetric about the y-axis), select **Reflections — *y*-axis** from the **Tools** menu. If the graph is its own reflection, then it is symmetric about the *y*-axis (at least on the plotted domain). To check if a function is an odd function (symmetric about the origin), select **Reflections — origin** from the **Tools** menu. If the graph is its own reflection, then it is symmetric about the origin (at least on the plotted domain).

Asymptotes

If **Adaptive Plot** in the **Graph** menu is checked, *TEMATH* will display the vertical asymptotes of the plotted function in the Graph window and it will write the values of

the vertical asymptotes into the Report window. To find horizontal and slant (oblique) asymptotes, select **Other Asymptotes** from the **Tools** menu.

Local Maximum and Minimum Values

To find local maximum and minimum values, click the **Maximum/Minimum Finder** tool  and click in the **Graph** window near the local maximum or minimum. The value and the coordinate location of the local maximum or minimum will be written into the Report window.

Concavity and Points of Inflection

Use the Rectangular Tracker tool  to approximate the points of inflection and the intervals over which the graph of the function is concave up or concave down.

Printing a Graph

To print a copy of a graph, click in the **Graph** window to make it active and select **Print Graph** from the **File** menu.

The Complete Graph

The graph of a function is *complete* if it shows all the important *features* of the graph and it suggests what the entire graph would look like if it were plotted over the function's entire domain. The complete graph should show (or suggest) all intercepts, local maximums and minimums, points of inflection, asymptotes, symmetry, discontinuities; and, it should suggest the end-behavior of the graph and the domain of the graph. The selection of the graph viewing window is very important and there are times when one viewing window is not sufficient and more than one is needed. As an example, consider the function $f(x) = x^3 - x$. If you plot $f(x)$ using *TEMATH*'s default domain $-5 \le x \le 5$ with the autoscaling option on, you get the graph shown in Figure 1. This graph suggests the end-behavior of the graph and the overall shape of the graph, but it gives very little information regarding the intercepts and local extremes. If you change the plotting domain to $-1.5 \le x \le 1.5$, you get the graph shown in Figure 2. This graph shows all the important features of $f(x)$ and it still suggests the graph's end-behavior.

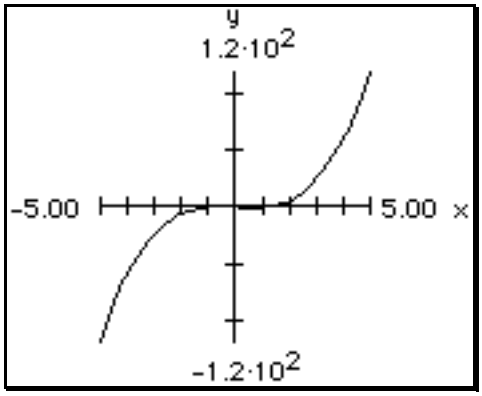

Figure 1

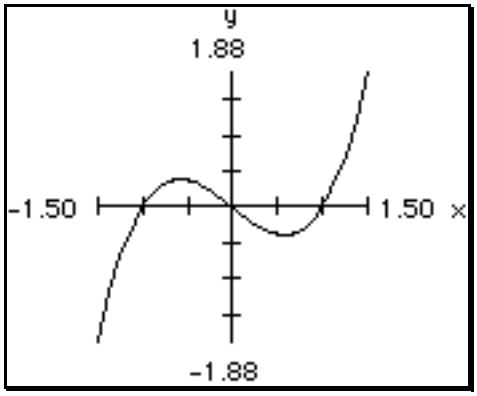

Figure 2

As another example, consider the function $f(x) = x^4 - \sqrt{x^2 - 1}$. Using the viewing window $-5 \le x \le 5$ and $0 \le y \le 500$, you get the graph shown in Figure 3. This graph shows the end-behavior but it gives no valuable information concerning the graph's domain or its other important features. If you change the viewing window to $-1.5 \le x \le 1.5$ and $0 \le y \le 2$, you get the graph shown in Figure 4. This graph shows that $f(x)$ has no intercepts, is not defined for $-1 < x < 1$, and has two local minimums.

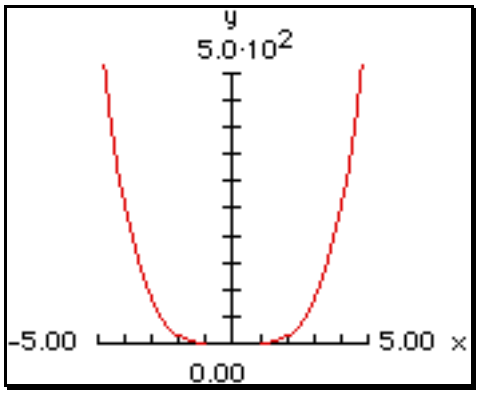

Figure 3

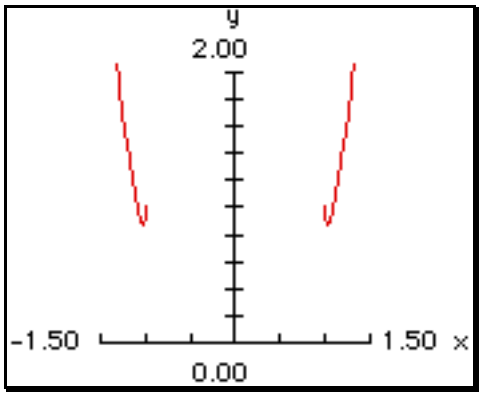

Figure 4

Note that you really need both of these graphs to get a complete picture of the graph of $f(x)$, in fact, you might even want the two additional graphs shown in Figures 5 and 6.

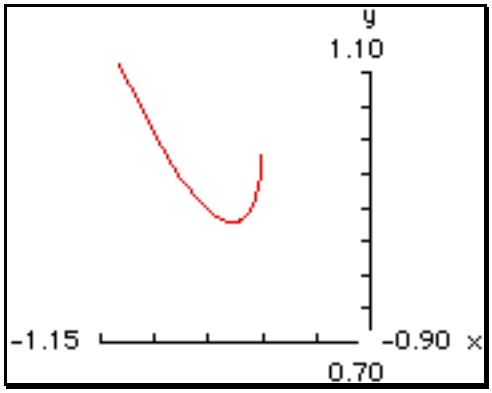

Figure 5

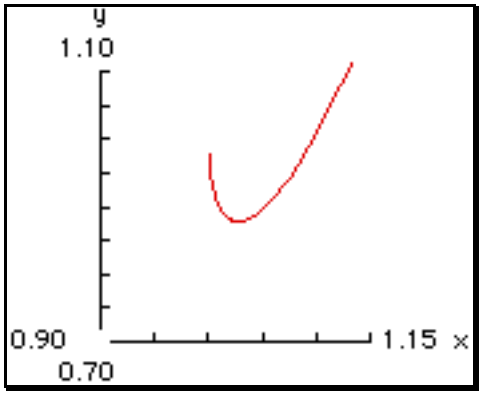

Figure 6

Exploration 1 Finding a Complete Graph

In the following problems, select an appropriate graph viewing window (or windows) that will give a complete graph of the given function. Carefully examine the function before selecting a viewing window. Can you guess what the shape of the graph will be? Can you determine the end-behavior of the graph? Can you estimate the domain and range of the function? Try a variety of graphing windows (both large and small) but use as few viewing windows as possible in your final graph. Attach to this lab a **printed**

copy of the complete graph for each of the functions. On each graph be sure to label the *x*-intercepts, the *y*-intercept, the local maximums and minimums, and all the asymptotes.

1. Plot the function $f(x) = x^2 - 103x + 52$. The *TEMATH* expression for this function is **x^2 – 103x + 52**.

   Domain of $f(x)$ ……Range of $f(x)$……

   x-intercepts …… *y*-intercept ……

   Asymptotes……

   Local maximum and minimum values ……

   ……

2. Plot the function $f(x) = x^3 - 2.1x^2 - 4x + 8.4$. The *TEMATH* expression for this function is **x^3 – 2.1x^2 – 4x + 8.4**.

   Domain of $f(x)$ ……Range of $f(x)$……

   x-intercepts …… *y*-intercept ……

   Asymptotes……

   Local maximum and minimum values ……

   ……

3. Plot the function $x^4 - \sqrt{1-x}$. The *TEMATH* expression for this function is **x^4 – sqrt(1 – x)**.

   Domain of $f(x)$ ……Range of $f(x)$……

   x-intercepts …… *y*-intercept ……

   Asymptotes……

   Local maximum and minimum values ……

   ……

4. Plot the function $f(x) = \frac{x}{x^{1/3} + 5}$. The *TEMATH* expression for this function is **x/(rad(3, x) + 5)**. Note that $(g(x))^{1/n}$ can be written in *TEMATH* as rad(n, g(x)).

   Domain of $f(x)$ ……Range of $f(x)$……

   x-intercepts …… *y*-intercept ……

   Asymptotes……

   Local maximum and minimum values ……

   ……

5. Plot the function $f(x) = \frac{x+1}{\sqrt{x^2 - 10x + 16}}$. The *TEMATH* expression for this function is **(x + 1)/sqrt(x^2 – 10x + 16)**.

   Domain of $f(x)$ ……Range of $f(x)$……

   x-intercepts …… *y*-intercept ……

   Asymptotes……

   Local maximum and minimum values ……

   ……

6. Plot the function $f(x) = 2^{x/10} - x^2 - 0.9$. The *TEMATH* expression for this function is **2^(x/10) – x^2 – 0.9**.

   Domain of $f(x)$ ……Range of $f(x)$……

   x-intercepts …… *y*-intercept ……

   Asymptotes……

   Local maximum and minimum values ……

   ……