Name________________________________ **Student ID #** _________________________
Instructor____________________________ **Lab Period** _____ **Date Due** ___________

Lab 2
Power Functions

Objectives

1. To become familiar with the definition of a power function, an even function, an odd function, and a root function.
2. To examine some of the properties of the graphs of power functions.

A *power function* is defined as $f(x) = x^a$, where a is a constant.

Exploration 1 Power Functions with Integer Powers

Let's start this exploration by examining the graphs of power functions for positive, even, integer powers. First, be sure that autoscaling is turned *on*, then, enter and plot the function $f(x) = x^2$ on the interval $\mathbf{-3 \le x \le 3}$. Next, overlay the plot of the function $f(x) = x^4$. Finally, overlay the plot of the function $f(x) = x^6$.

1. Are all these graphs symmetric about the *x*-axis, the *y*-axis, or the origin? ………………

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Now, we will examine combinations of these power functions. Enter and plot the functions: $f(x) = x^2 + x^4$, $f(x) = x^2 - x^4$, $f(x) = x^2 + x^4 + x^6$, and $f(x) = x^2 - x^4 + x^6$ on $-3 \le x \le 3$.

2. Are all these graphs symmetric about the *x*-axis, the *y*-axis, or the origin? ………………

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3. Show algebraically that $f(-x) = f(x)$ for all these functions.………………………………

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> A function f that satisfies $f(-x) = f(x)$ for all values of x in its domain is called an *even function*. The graph of an even function is symmetric about the y-axis.

In the next part of this exploration, we will examine power functions for positive, odd, integer powers. To begin, let's enter and plot the function $f(x) = x$ on the interval $-3 \le x \le 3$. Next, overlay the plots of the functions $f(x) = x^3$ and $f(x) = x^5$.

4. Are all these graphs symmetric about the x-axis, the y-axis, or the origin? ………………

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   What happens if we form new functions by adding or subtracting these power functions? To find out, enter and plot the functions: $f(x) = x - x^5$, $f(x) = x^3 + x^5$, $f(x) = -x + x^3$, and $f(x) = -x - x^3 + x^5$ on $-3 \le x \le 3$.

5. Are all these graphs symmetric about the x-axis, the y-axis, or the origin? ………………

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6. Show algebraically that $f(-x) = -f(x)$ for all these function. ……………………………

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> A function f that satisfies $f(-x) = -f(x)$ for all values of x in its domain is called an *odd function*. The graph of an odd function is symmetric about the origin.

Finally, let's examine functions containing both even and odd positive integer powers. Enter and plot the functions: $f(x) = x + x^2$, $f(x) = x^2 + x^3$, $f(x) = x - x^2 + x^3$, and $f(x) = -x + x^4$ on $-3 \le x \le 3$.

7. a) Are any of these graphs symmetric about the x-axis, the y-axis, or the origin?.………

   b) Are any of these functions even or odd?…………. Explain.…………………………

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8. If we form a function by adding a constant to an even function, for example, $f(x) = 2 + x^2 + x^4$, is this new function also even? Give reasons for your answer………

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9. If we form a function by adding a constant to an odd function, for example, $f(x) = 4 + x - x^3$, is this new function also odd? Give reasons for your answer..……….

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10. Give the general form of all polynomials that are even functions. ……………………….

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11. Give the general form of all polynomials that are odd functions..………………………..

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The power function for $a = 1/n$, where n is a positive integer, is called the *root function*. *TEMATH* has a specially predefined root function (or radical function) for computing the n^{th} root of x, $\sqrt[n]{x} = x^{1/n}$. This predefined function is **rad(*n*, *x*)**. It is always best to use rad(n, x) to compute $x^{1/n}$ since the computer treats the expression x^(1/n) as undefined for $x < 0$.

Exploration 2 Power Functions with Fractional Powers

The purpose of this exploration is to examine some of the properties of the root function. Let's begin by doing the following:

- Click in the **Domain & Range** window and turn autoscaling *off*.
- Enter the plotting domain $-2 \le x \le 2$ and the plotting range $-2 \le y \le 2$.
- Enter and plot the function $f(x) = x$.
- Enter and overlay the plot of $f(x) = x^{1/2} = \sqrt{x}$. In *TEMATH*, $x^{1/2}$ is written as **sqrt(x)** or **rad(2, x)**.
- Enter and overlay the plot of $f(x) = x^{1/3} = \sqrt[3]{x}$. In *TEMATH*, $x^{1/3}$ is written as **rad(3, x)**.
- Enter and overlay the plot of $f(x) = x^{1/4} = \sqrt[4]{x}$.
- Enter and overlay the plot of $f(x) = x^{1/5} = \sqrt[5]{x}$.

1. a) What is the domain of a root function that is an odd root of x?………………………

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b) What is the domain of a root function that is an even root of x? ……………………

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2. a) What coordinates do all these graphs have in common? ……………………………

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b) Explain why all these graphs pass through these coordinates..………………………

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c) What additional coordinates do all the graphs of the odd root functions have in common?…………………………………………………………………………

3. Describe what happens to these graphs as the fractional power changes from $\frac{1}{2}$ to $\frac{1}{5}$ (consider the intervals $0 < x < 1$ and $x > 1$). Give reasons for your answer…………….
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4. Give a list of the graphical properties that all these root functions have in common.…….
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