Name________________________________ **Student ID #** _________________________
Instructor____________________________ **Lab Period** _____ **Date Due** ___________

Lab 1
Polynomials

Objectives

1. To become familiar with the definition of a polynomial.
2. To examine some of the properties of the graphs of polynomials.
3. To conjecture a general result from a set of examples.

TEMATH Procedures

Enter and Plot a Function

To enter a function,

- Select **New Function** from the **Work** menu.
- Enter the expression for the function into the first cell of the **Work** window. When finished, click the **Enter** button in the **Work** window or press the **Enter** or the **Return** key.

To plot a function,

- Enter the plotting domain of the function into the **Domain & Range** window.
- Select **Plot** from the **Graph** menu.

Enter and Overlay the Graph of a Function

To enter a function and overlay its graph on top of an existing graph,

- Enter the function into the **Work** window.
- Select **Overlay** from the **Graph** menu.

Change the Plot Pattern or Color

To change the plot pattern or color,

- Click the pattern or color to the left of the function in the **Work** window until you get the one that you want.
- Plot the function.

Set the Plotting Domain

To set the plotting domain of subsequent plots,

- Click in the **Domain & Range** window to make it active.

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- Enter the minimum and maximum values of the plotting domain that you want into the cells to the left and to the right of ≤ **x** ≤. Note that *TEMATH* uses $-5 \le x \le 5$ as the default for the plotting domain.

Turn Off/On Autoscaling

Autoscaling is *on* when the autoscaling box in the **Domain and Range** window contains an × and it is *off* when there is no ×. Click in the **Domain & Range** window to make it active and then click the **autoscaling box** to display the × or to remove the ×.

Use the Root Finder Tool to Find a Real Root of a Function

To find a real root of the plotted function,

- Click the **Root Finder** tool on the tool palette in the **Graph** window to make it active.
- Click in the **Graph** window near the root. *TEMATH* will find the root and write its value into the Report window.

A *polynomial* is a function of the form

$$p(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$$

where n is a nonnegative integer called the *degree* of the polynomial and $a_0, a_1, a_2, ..., a_n$ are constants called the *coefficients* of the polynomial. Polynomials are very useful for modeling physical phenomena and for approximating complicated functions.

Exploration 1 Roots (zeros) of a Polynomial

A real *root* or *zero* of a polynomial is a real number r such that $p(r) = 0$. Graphically, this is where the graph of the polynomial crosses or touches the x-axis. You can think of these points as the x-intercepts of the graph.

In this exploration, we want to investigate the different possibilities for the roots of a cubic polynomial. Let's start by entering and plotting the third degree (cubic) polynomial $p(x) = -9 - 9x + x^2 + x^3$ on the interval $-5 \le x \le 5$. Note that you can enter the function expression **– 9 – 9x + x^2 + x^3** or you can use the *TEMATH* predefined polynomial function **poly(x, – 9, – 9, 1, 1)**. To find a root of the polynomial, click the **Root Finder** tool and then click in the **Graph** window near the root. *TEMATH* will find the root and write its value into the Report window.

1. What are the values of the three real roots of this polynomial?................................

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End Behavior

The $a_n x^n$ term of a polynomial dominates $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$ as $|x|$ gets large because $\left|a_n x^n\right|$ becomes larger than the absolute value of any other term in the polynomial. Thus, the value of $a_n x^n$ as x becomes much larger than zero determines the *far-right behavior* of the polynomial and the value of $a_n x^n$ as x becomes much smaller than zero determines the *far-left behavior* of the polynomial. Since $x^n > 0$ whenever $x > 0$, then $a_n x^n > 0$ whenever $a_n > 0$ and the far-right behavior of the polynomial is said to be *up to the right* ↗; otherwise, whenever $a_n < 0$, then $a_n x^n < 0$ and the far-right behavior of the polynomial is said to be *down to the right* ↘. Similarly, if $a_n x^n > 0$ whenever $x < 0$, then the far-left behavior of the polynomial is said to be *up to the left* ↖, and, if $a_n x^n < 0$ whenever $x < 0$, then the far-left behavior of the polynomial is said to be *down to the left* ↙.

2. Describe the end-behavior of the polynomial $p(x) = -9 - 9x + x^2 + x^3$ ....

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If a cubic polynomial has the three distinct real roots r_1, r_2, and r_3, then the polynomial can be written in factored form as $p(x) = a(x - r_1)(x - r_2)(x - r_3)$. If $a > 0$, then the end behavior of the polynomial is *down to the left* and *up to the right*, otherwise, it is *up to the left* and *down to the right.*

3. a) What is a for $p(x) = 12 - 10x - 4x^2 + 2x^3$? ....

b) What is the factored form of $p(x) = 12 - 10x - 4x^2 + 2x^3$?...............................

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Enter and plot this factored form of $p(x)$ on the interval $-5 \le x \le 5$. Now, let's gain some intuition about the factored form by doing the following:

- Edit the factored form of $p(x)$ by changing the value of a to 1. Remember to press the Enter or Return key when you are finished.
- Change the plot color/pattern of this polynomial by clicking on the vertical line immediately to the left of the polynomial in the Work window. This will help you to distinguish the different polynomials that will be plotted in the Graph window.
- Select **Overlay Plot** from the **Graph** menu.

Repeat the above sequence of instructions for $a = 3$, –1, –2, and –3.

4. Describe what happens to the graphs of the polynomials as the value of *a* changes. …….

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As a second example, enter and plot the polynomial $p(x) = -8 - 4x + 2x^2 + x^3$ on the interval $-5 \le x \le 5$. This time, find all the roots of the polynomial in its plotted domain by selecting **Root Finder** from the **Tools** menu. The values of all the roots will be written into the Report window.

5. a) What are the values of all the real roots of this polynomial? …………………………

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b) What is different about the graph of this polynomial near one of its roots? …………

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If a polynomial has a factored form $p(x) = a(x - r_1)(x - r_2)^2$, then we will call r_1 a *simple* root and r_2 a *multiple root of multiplicity two*. If the graph of a polynomial touches the *x*-axis at $x = r$, then r is a multiple root.

6. What is the factored form of $p(x) = -8 - 4x + 2x^2 + x^3$?…………………………………

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As a final example, enter and plot the polynomial $p(x) = 4 + 4x + x^2 + x^3$ on the interval $-5 \le x \le 5$.

7. a) What are the values of all the real roots of this polynomial? …………………………

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b) What is the factored form of $p(x) = 4 + 4x + x^2 + x^3$ (not all factors are linear)?……

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c) Why can't the quadratic term be factored into the product of two linear factors?…….

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9. Is it possible for a cubic polynomial to have no real roots? ……… Explain your answer.

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Exploration 2 Roots and the Degree of a Polynomial

Your goal in this exploration is to discover the relationship between the number of real roots of a polynomial and the degree of the polynomial. To help you discover this relationship, you will need to complete the following table. To find the values you need to fill-in this table, first turn *off* autoscaling and set the domain to $\mathbf{-3 \le x \le 3}$ and set the range to $\mathbf{-5 \le y \le 5}$. Then, enter and plot the first polynomial from the table below. Remember, in *TEMATH*, the polynomial $p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$ can be entered as $poly(x, a_0, a_1, a_2, \ldots, a_n)$. For example, the polynomial $p(x) = 1 + x^2 = 1 + 0x + 1x^2$ can be entered into *TEMATH* as **poly(*x*, 1, 0, 1)**. Next, select **Root Finder** from the **Tools** menu. *TEMATH* will find all the real roots in the plotted domain and write their values into the Report window. Finally, write the degree of the polynomial, the values of all the real roots of the polynomial, and the number of real roots of the polynomial into the table given below. Repeat this process for all the polynomials given in the following table.

Polynomial	Degree of Polynomial	Number of Real Roots	Values of Real Roots
$p(x) = 1 + x^2$			
$p(x) = 1 + 2x + x^2$			
$p(x) = -1 + x^2$			
$p(x) = 1 + x^3$			
$p(x) = -x + x^3$			
$p(x) = 3x - 6x^2 + 3x^3$			
$p(x) = 1 + x^4$			
$p(x) = 1 - 4x + 6x^2 - 4x^3 + x^4$			
$p(x) = 1 - 2x^2 + x^4$			
$p(x) = -x^2 + x^4$			
$p(x) = 2x - x^2 - 2x^3 + x^4$			

1. Based on the results in the above table, what can you conclude about the relationship between the number of real roots of a polynomial and the degree of the polynomial?…..

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