

## Tips for Success with Proofs and Disproofs

Make sure your proofs are genuinely convincing. Express yourself carefully and completely – but concisely! Write in complete sentences, but don't use an unnecessary number of words.

### Disproof by Counterexample

- To disprove a universal statement, give a counterexample.
- Write the word “Counterexample” at the beginning of a counterexample.
- Write counterexamples in complete sentences.
- Give values of the variables that you believe show the property is false.
- Include the computations that prove beyond any doubt that these values really do make the property false.

### All Proofs

- Write the word “Proof” at the beginning of a proof.
- Write proofs in complete sentences.
- Start each sentence with a capital letter and finish with a period.

### Direct Proof

- Begin each direct proof with the word “Suppose.”
- In the “Suppose” sentence:
  - Introduce a variable or variables (indicating the general set they belong to - e.g., integers, real numbers etc.), and
  - Include the hypothesis that the variables satisfy.
- Identify the conclusion that you will need to show in order to complete the proof.
- Reason carefully from the “suppose” to the “conclusion to be shown.”
- Include the little words (like “Then,” “Thus,” “So,” “It follows that”) that make your reasoning clear.
- Give a reason to support each assertion you make in your proof.

### Proof by Contradiction

- Begin each proof by contradiction by writing “Suppose not. That is, suppose...,” and continue this sentence by carefully writing the negation of the statement to be proved.
- After you have written the “suppose,” you need to show that this supposition leads logically to a contradiction.
- Once you have derived a contradiction, you can conclude that the think you supposed is false. Since you supposed that the given statement was false, you now know that the given statement is true.

### Proof by Contraposition

- Look to see if the statement to be proved is a universal conditional statement.
- If so, you can prove it by writing a direct proof of its contrapositive.