Some Mathematical Conventions

1. When introducing a new variable into a discussion, the convention is to place the new variable to the left of the equal sign and the expression that defines it to the right. This convention is identical to the one used in computer programming. For example, in a computer program, if a and b have previously been defined, and you want to assign the value of $a + b$ to a new variable s, you would write something like

$$s := a + b.$$

Similarly, in a mathematical proof, if a and b have previously been introduced into a discussion, and you want to let s be their sum,

instead of writing "Let $a + b = s$," you should write, "Let $s = a + b$."

2. It is considered good mathematical writing to avoid starting a sentence with a variable. That is one reason that mathematical writing frequently uses words and phrases such as Then, Thus, So, Therefore, It follows that, Hence, etc. For example, in a proof that any sum of even integers is even, instead of writing,

By definition of even, $m = 2a$ and $n = 2b$ for some integers a and b.

$$m + n = 2a + 2b \ldots$$

write

By definition of even, $m = 2a$ and $n = 2b$ for some integers a and b.
Then

$$m + n = 2a + 2b \ldots$$

The fact that $m+n = 2a+2b$ is a consequence of the facts that $m = 2a$ and $n = 2b$. Including the word "Then" in your proof alerts your reader to this reasoning.

3. Standard mathematical writing avoids repeating the left-hand side in a sequence of equations in which the left-hand side remains constant. For example, if $n = 5q + 4$, instead of writing

$$\begin{aligned} n^2 &= (5q + 4)^2 \\ n^2 &= 25q^2 + 40q + 16 \\ n^2 &= 25q^2 + 40q + 15 + 1 \\ n^2 &= 5(5q^2 + 8q + 3) + 1 \end{aligned}$$

all the n^2 except the first are omitted and each subsequent equal sign is read as "which equals," as shown below:

$$\begin{aligned} n^2 &= (5q + 4)^2 \\ &= 25q^2 + 40q + 16 \\ &= 25q^2 + 40q + 15 + 1 \\ &= 5(5q^2 + 8q + 3) + 1 \end{aligned}$$

4. Respecting the equal sign is one of the most important mathematical conventions. An equal sign should only be used between quantities that are equal, not as a substitute for words like "is," "means that," "if and only if," $\Leftrightarrow$, or "is equivalent to." For example, if $a = 4$ and $b = 12$, students occasionally write:

$$a \mid b = 4 \mid 12 \text{ since } 12 = 4 \cdot 3.$$

But if this were read out loud, it would be, "a divides b equals 4 divides 12 since 12 equals 4 times 3," which makes no sense. A correct version would be

$$a \mid b \;\Leftrightarrow\; 4 \mid 12, \text{ which is true because } 12 = 4 \cdot 3$$

or

$$a \mid b \text{ because } 4 \mid 12 \text{ since } 12 = 4 \cdot 3.$$

5. It is unnecessary, and even risky, to place full statements of definitions and theorems inside the bodies of proofs. The reason is that the variables used to express them can become confused with variables that are part of the proof. So instead of including the statement of the definition of divisibility, for example, just write, "by definition of divisibility." Similarly, instead of including the statement of, say, Theorem 8.4.3, just write, "by Theorem 8.4.3." For instance, to prove that a sum of any even integer plus any odd integer is odd, someone might write the following:

   > Suppose m is any even integer and n is any odd integer.
   > For an integer to be even means that it equals $2k$ for some integer k, and
   > for an integer to be odd, means that it equals $2k+1$ for some integer k.
   > Thus $m + n = 2k + (2k + 1) = 4k + 1$...

   The problem is that although the letter k appears in the statements of the definitions in the text, it refers to a different quantity in each one. However, when the statements are combined together in the proof, the letter k can have only one interpretation. The result is that the argument in the "proof" only applies to an even integer and the next successive odd integer, not to *any* even integer and *any* odd integer.