MTH 181 **(1 pt) Section: Noon or 1 PM** **Name Solution Key**

Worksheet 0: Common Algebra Errors V2

This worksheet is intended to help with your ability to correct common algebra errors! Although, I will record your score, I will ***not*** include your score in your class average. Instead, your score on this worksheet will help me (and you) determine if you need to seek tutoring from the *Math-Business Tutoring Center* located in *Room 001 LARTS* (basement floor).

For each case, present a *clear justification* as to why the expressions on each side of the equals sign are *unequal*.

a) (2 pt) $\sqrt{x^2+9} \neq x+3$, **Why?**

Solution:
Cause: The *mistaken belief* that square root function *distributes* over addition. ***It doesn't!*** Here is why:

If $\sqrt{x^2+9} = x+3$ is *true*, then it must be true for *all* values of x! But

Method 1: The statement is *false* when $x=-3$: $\sqrt{(-3)^2+9} = \sqrt{18} \neq 0 = -3+3$.

Method 2: $\sqrt{x^2+9} = x+3 \Rightarrow x^2+9 = (x+3)^2 = x^2+6x+9 \Rightarrow x=0$ is the only value for which the statement is true!

□

b) (2 pt) $\dfrac{x^2+1}{x} \neq x+1$, **Why?**

Solution:
Cause: The *mistaken belief* that x in the denominator can be divided out of x^2 in the numerator. *It can't!* Here's why:

If $\dfrac{x^2+1}{x} = x+1$ is *true*, then it must be true for *all* values of x! But

Method 1: The statement is *false* when $x=0$: $\dfrac{0^2+1}{0} = \dfrac{1}{0} = \textit{undefined} \neq 1 = 0+1$.

Method 2: $\dfrac{x^2+1}{x} = x+1 \Rightarrow x^2+1 = x^2+x \Rightarrow x=1$ is the only value for which the statement is true!

Method 3: $\dfrac{x^2+1}{x} = \dfrac{x^2}{x} + \dfrac{1}{x} = x + \dfrac{1}{x} \neq x+1$ since $\dfrac{1}{x} \neq 1$ by correctly using the rule for adding rational expressions.

□

c) (2 pt) $\dfrac{n}{n+1} \neq 1+n$, **Why?**

Solution:
Cause: The *mistaken belief* that the rational expression $\dfrac{n}{n+1}$ can be split into the sum $\dfrac{n}{n} + \dfrac{n}{1} = 1+n$. It can't! Here's why:

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If $\dfrac{n}{n+1} = 1 + n$ is *true*, then it must be *true* for *all* values of n! But

Method 1: The statement is *false* when $n = 0$: $\dfrac{0}{0+1} = 0 \neq 1 = 0 + 1$.

Method 2: $\dfrac{n}{n+1} = n+1 \Rightarrow n = (n+1)^2 \Rightarrow n^2 + n + 1 = 0 \Rightarrow (n+1/2)^2 + 3/4 = 0$ which can't be true for any real number; that is, the statement is only true when $n = \left(-1+\sqrt{3}i\right)/2$ or $n = \left(-1-\sqrt{3}i\right)/2$ (complex numbers).

□

d) (2 pt) $1 - \dfrac{x-1}{2} \neq \dfrac{1-x}{2}$, **Why?**

Solution:

Cause: The *failure* to distribute the minus sign over a *difference of terms* correctly. Here's how to check:

If $1 - \dfrac{x-1}{2} = \dfrac{1-x}{2}$ is *true*, then it must be true for *all* values of x! But

Method 1: The statement is *false* when $x = 0$: $1 - \dfrac{0-1}{2} = 1 + \dfrac{1}{2} = \dfrac{3}{2} \neq \dfrac{1}{2} = \dfrac{1-0}{2}$.

Method 2: $1 - \dfrac{(x-1)}{2} = \dfrac{2 + (-1)(x-1)}{2} = \dfrac{2 - x + 1}{2} = \dfrac{3-x}{2} = \dfrac{3}{2} - \dfrac{x}{2}$ but $\dfrac{1-x}{2} = \dfrac{1}{2} - \dfrac{x}{2}$ by correctly applying rules for combining rational expressions.

□

e) (2 pt) $-x^2 \neq (-x)^2$, **Why?**

Solution:

Cause: An *order of operations error*!

The *exponentiation* operator is evaluated *before* the *subtraction* operator; thus,

$$-x^2 = -(x^2) \leq 0$$

But

$$(-x)^2 = x^2 \geq 0.$$

Therefore, $-x^2 = -(x^2) \neq (-x)^2 = x^2$ unless $x = 0$.

□

f) (2 pt) $x^2 = 1 \not\Rightarrow x = 1$, **Why?**

Solution:

Cause: The *failure* to recognize $x^2 = 1$ to be a *quadratic* equation. Here is how to treat $x^2 = 1$ correctly!

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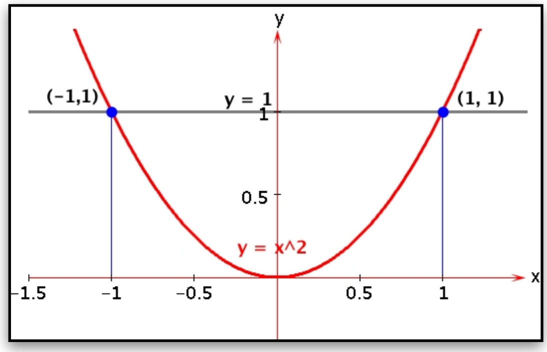
Method 1: Observe that $x^2 = 1 \Leftrightarrow x^2 - 1 = 0 \Leftrightarrow (x+1)(x-1) = 0$ which has both $x = -1$ and $x = 1$ as solutions!

Thus, $x^2 = 1 \not\Rightarrow x = 1$ since x may equal -1 instead of 1. The correct statement is

$$x^2 = 1 \Rightarrow x = 1 \text{ or } x = -1 .$$

Method 2: Plot both $y = x^2$ and $y = 1$ on the same graph:

The graph shows that $x^2 = 1$ has both $x = -1$ and $x = 1$ as solutions



□

g) (2 pt) $(4x-1)^2 \neq 16x^2 + 1$, **Why?**

Solution:

Cause: The *mistaken belief* that exponentiation *distributes* over addition. **It doesn't!** Here is why:

If $(4x-1)^2 = 16x^2 + 1$ is *true*, then it must be true for *all* values of x! But

Method 1: The statement is *false* when $x = 1$: $(4 \cdot 1 - 1)^2 = 9 \neq 17 = 16(1)^2 + 1$.

Method 2: If $(4x-1)^2 = 16x^2 + 1$ then $16x^2 - 8x + 1 = 16x^2 + 1 \implies x = 0$ is the only value for which the statement is true!

□

h) (2 pt), $3 \cdot k^4 \neq (3k)^4$, **Why?**

Solution:

Cause: An *order of operations error*! Exponentiation is evaluated *before* multiplication! Here's why the evaluation order of exponentiation and multiplication matters:

If the statement $3 \cdot k^4 = (3k)^4$ is *true*, it must be true for *all* values of k! But

Method 1: The statement is *false* when $k = 1$: $3 \cdot (1)^4 = 3 \neq 81 = (3 \cdot 1)^4$.

Method 2: $3 \cdot k^4 = (3k)^4 \Rightarrow 3 \cdot k \cdot k \cdot k \cdot k = 3k \cdot 3k \cdot 3k \cdot 3k \Rightarrow 3 \cdot k \cdot k \cdot k \cdot k = 81 \cdot k \cdot k \cdot k \cdot k$

$\Rightarrow 0 = 78 \cdot k \cdot k \cdot k \cdot k \Rightarrow k = 0$

is the only value for which the statement is *true*.

□

i) (2 pt) $n^2 \geq 4 \not\Rightarrow n \geq 2$, **Why?**

Solution:

Cause: The *failure* to recognize $n^2 \geq 4$ to be a *quadratic inequality*,. Here is how to treat $n^2 \geq 4$ correctly!

Method 1: Observe that $n^2 \geq 4 \Leftrightarrow n^2 - 4 \geq 0 \Leftrightarrow (n+2)(n-2) \geq 0$.

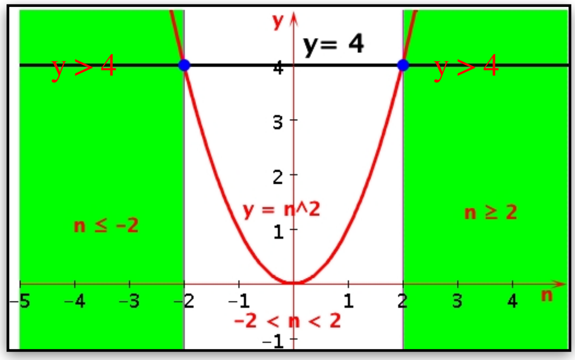
But this can only be true when either

$$(1)\ n+2\geq 0 \text{ and } n-2\geq 0 \quad \text{or} \quad (2)\ n+2\leq 0 \text{ and } n-2\leq 0.$$

Now, (1) implies $n\geq -2$ and $n\geq 2$; hence, $n\geq 2$. But (2) implies $n\leq -2$ and $n\leq 2$; thus, $n\leq -2$. Therefore, the correct statement is

$$n^2\geq 4 \Rightarrow n\leq -2 \text{ or } n\geq 2.$$

Method 2: Plot both $y=n^2$ and $y=4$ on the same graph: The graph shows that $n^2\geq 4$ is true when either $n\leq -2$ or $n\geq 2$ as solutions!



□

j) (2 pt) $x>0 \not\Rightarrow x^2>x$, **Why?**

Solution:

Cause: Using *only* positive integers (or real numbers ≥ 1) to test conjectures! Here's how to show $x>0 \not\Rightarrow x^2>x$.

Method 1: If $x=1/2$ then $x^2=1/4<1/2=x$. So it is possible for $x^2<x$ to be *true* with $x>0$. Therefore, $x>0$ doesn't imply that $x^2>x$.

Method 2: Assume both $x>0$ and $x^2>x$. Then both sides of the inequality $x^2>x$ can be divided by x without changing the direction of the inequality! Thus, $x>1$. Therefore the correct statement is

$$x>1 \Rightarrow x^2>x.$$

□

k) (2 pt) $\sqrt{x^2}\neq x$ (there was a typo in the original version of the worksheet) **Why?**

Solution:

Cause: Forgetting that x can have a negative value! Here is why this causes the error!

If $\sqrt{x^2}=x$ is *true*, it must be true for *all* values of x! But

Method 1: The statement is *false* when $x=-1$: $\sqrt{(-1)^2}=\sqrt{1}=1\neq -1$.

Method 2: Notice that $\sqrt{x^2}=|x|\geq 0$; thus, if $x<0$, $\sqrt{x^2}\neq x$. The correct statement is $\sqrt{x^2}=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x<0 \end{cases}$

□

l) (2 pt) $\frac{0}{10}\neq \textit{undefined}$, **Why?**

Solution:

Cause: Mixing up the expressions $\frac{0}{10}$ and $\frac{10}{0}$.

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Observe that $\frac{0}{10}=0$ since $0=0\cdot 10$. The correct statement is

$$\frac{10}{0}=undefined$$

since division is equivalent *repeated subtraction* and repeatedly subtracting 0 from 10 will *never* make 10 smaller!

□