

1. Modus Ponens $p \rightarrow q$ p $\therefore q$	2. Modus Tollens $p \rightarrow q$ $\sim q$ $\therefore \sim p$	3. Generalizations a) p $\therefore p \vee q$ b) q $\therefore p \vee q$
4. Specialization a) $p \wedge q$ p $\therefore q$ b) $p \wedge q$ q $\therefore p$	5. Conjunction p q $\therefore p \wedge q$	6. Elimination a) $p \vee q$ $\sim q$ $\therefore p$ b) $p \vee q$ $\sim p$ $\therefore q$
7. Transitivity $p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	8. Proof by Division into Cases $p \vee q$ $p \rightarrow r$ $q \rightarrow r$ $\therefore r$	9. Contradiction Rule $\sim p \rightarrow c$ $\therefore p$

Example: Given the following, where are my glasses?

- a) If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.
- b) If my glasses are on the kitchen table, then I saw them at breakfast.
- c) I did not see my glasses at breakfast.
- d) I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.
- e) If I was reading the newspaper in the living room, then my glasses are on the coffee table.

Let RK = "I was reading the newspaper in the kitchen."

GK = "My glasses are on the kitchen table."

SB = "I saw my glasses at breakfast."

RL = "I was reading the newspaper in the living room."

GC = "My glasses are on the coffee table."

then the natural language statements above can be expressed as the following formal statements:

$$a') RK \rightarrow GK \quad b') GK \rightarrow SB \quad c') \sim SB \quad d') RK \vee RL \quad e') RL \rightarrow GC$$

Now the following argument shows that my glasses are on the coffee table:

Step 1: Transitivity $RK \rightarrow GK$ (a') $GK \rightarrow SB$ (b') $\therefore RK \rightarrow SB$	Step 2: Modus Tollens $RK \rightarrow SB$ (1) $\sim SB$ (c') $\therefore \sim RK$	Step 3: Elimination $RK \vee RL$ (d') $\sim RK$ (2) $\therefore RL$	Step 4: Modus Ponens $RL \rightarrow GC$ (e') RL (3) $\therefore GC$
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Thus, by **Step 4** my glasses are on the coffee table!