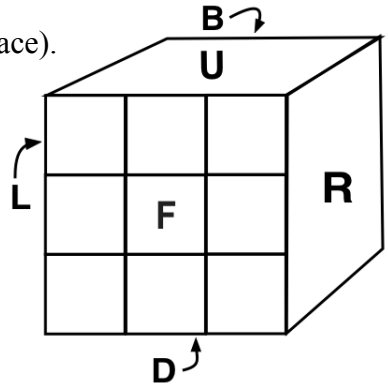
Here is a solution for the Rubik's Cube based on the solution found in David Singmaster's *Notes on the Magic Cube*, Enslow Publishers, 1981. In the following, **unlike** the usual mathematical convention, ***F***, ***B***, ***L***, ***R***, ***U***, ***D*** denote ***clockwise*** rotations of the front, back, left, right, up, and down faces respectfully by 90 degrees. Similarly, $F^{-1}, B^{-1}, L^{-1}, R^{-1}, U^{-1}, D^{-1}$ denote ***counterclockwise*** rotations of the corresponding faces by 90 degrees. Also, the composition of rotations such as RFR^{-1} are applied from *left to right* instead right to left. Thus, RFR^{-1} means apply ***R,*** then ***F*** and finally R^{-1}.



Step 1: Put the edges in place for one of the faces; say, **U**-face (the top face).
Suppose we want to put the **UF**-piece (upper front edge) in place.

a) Why can we assume it is ***not*** in the **U**-face?
   Answer: If it is, then apply X^2 to side face ***X*** containing this piece which moves it to the **D**-face.

b) What moves should we use if it is in the **D**-face (the down face).
   Case 1: The **U** side of the **UF**-piece is down.
   Turn the **D**-face until the **UF**-piece is in the **F**-face, then apply F^2.
   Case 2: The **F**-side of the **UF**-piece is down.
   Turn the **D**-face until the **UF**-edge is in the **F**-face, then apply $F^{-1}U^{-1}RU$.

c) What should we do if it is in the *middle* layer?
   Answer: Then turn the middle layer so that the **UF** piece is in the **F**-face with the **F**-face forward. There are two cases:
   Case 1: The **UF**-piece is at the **FL**-position. Apply ***U***.
   Case 2: The **UF**-piece is at the **FR**-position. Apply U^{-1}.

By rotating the *cube* around its vertical axis as needed, and applying the steps above, all the edges can be put into place in the **U**-face.

Step 2: Put the corners in place in the **U**-face.
Suppose we want to put the **URF**-piece (top right front corner) place without disturbing any of the other **U**-face pieces.

a) Why can we assume it is ***not*** in the **U**-face?
   Answer: If it is, then apply ***X*** or X^3 to the side ***X*** containing the corner so that it is moved to the **D**-face with its **R**-side or **F**-side down.

b) Why can we assume it is at the **FRD**-position ?
   Answer: If it isn't rotate the **D**-face, so that it is!

At this point, the **URF**-piece is at the **FRD**-position and can have three different orientations: What moves should we use to put it in place if the **URF**-piece has the

Case 1: **FRD**-orientation (**U**-side front, **R**-side right, **D**-side down) ? Apply $D^{-1}R^{-1}DR$.
Case 2: **RDF**-orientation (**U**-side right, **R**-side down, **F**-side front) ? Apply $DFD^{-1}F^{-1}$.
Case 3: **DFR**-orientation (**U**-side down, **R**-side front, **F**-side right) ? Apply $FD^2F^{-1}D^2R^{-1}DR$.

By rotating the cube around its vertical axis as needed, and applying the steps above, all the corners can be put into place in the **U**-face with the correct orientation.

Now, turn the *cube* upside down so that the **D**-face and **U**-face are swapped. The **U**-face now contains all its correct corners but they are likely not to be in their correct locations or oriented correctly and may contain edge pieces that belong in the middle layer.

Step 3: Put the middle edges correctly in place.
Suppose we want to put an **FR**-piece into place without disturbing the down face or any other middle layer edge.

a) Why can we assume that it is in the **U**-face?
Answer: If it isn't, then turn the cube around its vertical axis so that this edge piece is at the **FR**-position. Next, apply $B^{-1}U\left(R^2U^2\right)^3U^{-1}B$ to move the piece to the **BU**-position. Finally, turn the cube back to its original position.

b) What moves should we apply if

Case 1: The **F**-side of the **FR**-edge piece is up?
First turn the **U**-face until the **FR**-piece is at the **UL**-position. Next, apply $LU\left(U^2F^2\right)^3U^{-1}L^{-1}$.

Case 2: The **R**-side of the **FR**-edge piece is up?
First turn the **U**-face until the **FR**-piece is at the **BU**-position. Next, apply $B^{-1}U\left(R^2U^2\right)^3U^{-1}B$.

At this point the down face and middle layer pieces are in place and the **U** face contains all its pieces but they may not be in their correct locations or have the correct orientations.

Step 4: Orient the **U**-edges correctly.
An even number of **U**-sides of the **U**-edge pieces will be up. By tuning the cube if necessary, you can assume one of the following is true:

a) The **UB**- and **UF**-pieces are incorrectly oriented. Apply $B\left(LUL^{-1}U^{-1}\right)B^{-1}$.

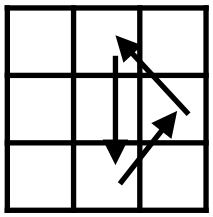
b) The **UB**- and **UL**-pieces are incorrectly oriented. Apply $B\left(ULU^{-1}L^{-1}\right)B^{-1}$.

c) All *four* edges are incorrectly oriented. Apply (a), turn the cube 180°, then apply (b).

Step 5: Put the U-edges in place.
Turn the **U**-face so that the **UF**-piece is at the **UF**-position. If a total of *two* edges are in place, apply U. You should now have 0, 1, or 4 **U**-face edges in place.

a) The edges are in place! Do nothing!

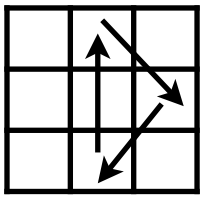
b) *One* edge is in place and the other three edges must be *cycled*.
Turn the *cube* so that the correct edge piece is at the **UL**-position



Counterclockwise

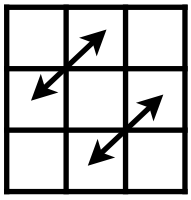
Case 1: The other three edges need to be cycled clockwise.
Apply $R^2D^{-1}\cdot U^2R^{-1}LF^2RL^{-1}\cdot DR^2$ (see the figure to the right).

Case 2: The other three edges need to be cycled counterclockwise.
Apply $R^2D^{-1}\cdot R^{-1}LF^2RL^{-1}U^2\cdot DR^2$ (see the figure to the left).



Clockwise

c) No edges are in place and two pairs of adjacent edges need to be swapped.
First turn the *cube* so that you need to swap the **U**-edge with the **UR**-edge and the **UL**-edge with the **UB**-edge. Apply $R^2D^2B^2D\left(L^2F^2\right)^3D^{-1}B^2D^2R^2$.



Swap Pairs

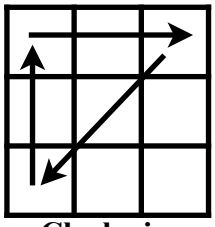
Step 6: Put the **U**-corners in place.

a) The corners are in place! Do nothing!

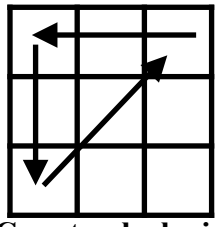
b) *One* corner is in place and *three* others need to be *cycled* into their correct positions.
Turn the *cube* so that the correct corner is at the **URF**-position

Case 1: The three other corners need to be cycled clockwise.
Apply $L^{-1} \cdot URU^{-1}R^{-1} \cdot L \cdot RUR^{-1}U^{-1}$ (see the figure on the left below).

Case 2: The *three* other corners need to be *cycled* counterclockwise.
Apply $URU^{-1}R^{-1} \cdot L^{-1} \cdot RUR^{-1}U^{-1} \cdot L$ (see the figure on the right below).

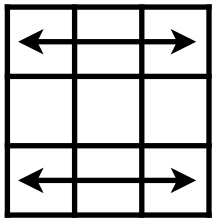


Clockwise

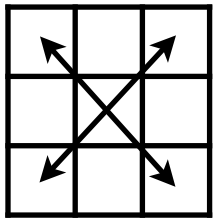


Counterclockwise

c) *No* **U**-corners are in place and adjacent corners need to be *swapped*.
Turn the **U**-face so the *swaps* are along the front and back edges of the **U**-face.
Apply $B\left(LUL^{-1}U^{-1}\right)^3 B^{-1}$ (see the figure on the left below).



Swap Adjacents



Swap Diagonals

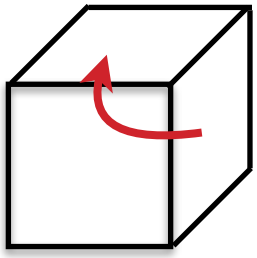
d) No **U** corners are in place and diagonal corners need to be swapped.
Apply $R^{-1}B^2\left(FRF^{-1}R^{-1}\right)^3 B^2R$ (see the figure on the right above).

Step 7: Orient the **U**-corners.

a) The **U**-corners are correctly oriented! Do nothing!

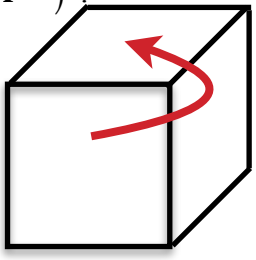
b) Turn the *cube* so that an incorrectly oriented corner is at the **URF**-position.
There are two cases.

Case 1: The corner piece needs to be rotated clockwise. Apply $\left(FDF^{-1}D^{-1}\right)^2$.



Case 2: The corner pice needs to be rotated counterclockwise. Apply $\left(DFD^{-1}F^{-1}\right)^2$.

Repeat (7b) for each incorrectly oriented corner.



The cube is now solved!