The order of the Rubik's Cube Group G_{Rubik} is calculated as follows:

First, notice that ignoring the orientation of the 27 pieces forming the cube, the eight corner pieces can be permuted among themselves and the 12 edge pieces can also be permuted among themselves.

Next, notice that each corner piece can be oriented in *three* ways and each edge piece can be oriented in *two* ways. This means that if we fix the orientation of a corner piece, the remaining 7 corner pieces can be oriented in $3^7 = 3^8/3$ ways. Similarly, if we fix the orientation of an edge piece, the remaining 11 edge pieces can be oriented in $2^{11} = 2^{12}/2$ ways.

Combining these observations, we see that $|G_{\mathbf{Rubik}}| = 8! \cdot 3^8/3 \cdot 12! \cdot 2^{12}/2$.

However, this is too large! Notice that rotating a face counterclockwise repeatedly, cycles the four corner pieces corners among them selves and cycles the four edge pieces among themselves. This means that we can express a face rotation as the product of two permutations

$$\begin{pmatrix} C_1 & C_2 & C_3 & C_4 \\ C_2 & C_3 & C_4 & C_1 \end{pmatrix}\begin{pmatrix} E_1 & E_2 & E_3 & E_4 \\ E_2 & E_3 & E_4 & E_1 \end{pmatrix}$$

where C_1, C_2, C_3, C_4 are consecutive labels for the face's corner pieces and E_1, E_2, E_3, E_4 are consecutive labels for the face's edge pieces (both ordered counterclockwise starting from the bottom-left corner of the face). Because of the form of these permutations, this product can be written as a product of two disjoint cycles

$$\begin{pmatrix} C_1 & C_2 & C_3 & C_4 \end{pmatrix}\begin{pmatrix} E_1 & E_2 & E_3 & E_4 \end{pmatrix}.$$

It is known that a cycle of even length is an *odd* permutation while a cycle of old length is an *even* permutation. This implies that a face rotation is equivalent to an even permutation and all the rotations of the faces form a group of even permutations (see the note below).

Note: A product of two even permutations or two odd permutations is also even while the product of an odd and an even permutation is an odd permutation. Because of this, the set of even permutations form a subgroup (known as the alternating subgroup) of all permutations. The order of this subgroup is exactly half of the number of all possible permutations (see Section 3.6 of Beachy and Blair and Chapter 5 of Judson).

This means that the true order of the Rubik's Cube group

$$|G_{\mathbf{Rubik}}| = \frac{1}{2}\left(8! \cdot \frac{3^8}{3} \cdot 12! \cdot \frac{2^{12}}{2}\right) = \frac{8! \cdot 3^8 \cdot 12! \cdot 2^{12}}{12} = 2^{27} \cdot 3^{14} \cdot 5^3 \cdot 7^2 \cdot 11$$
$$= 43252\,00327\,44898\,56000 \approx 4.3 \times 10^{19}.$$

The 12 in the denominator is the number of ways the Rubik's Cube can be assembled from its pieces; that is, if you assemble the cube randomly, then you have 1 chance in 12 of being able to restore the cube to a state where all the faces have a single color.

Reference: David Singmaster , *Notes on Rubik's Magic Cube*, 1981, Enslow Publishers .